Beam-beam Effects and Luminosity Evolution in Tevatron

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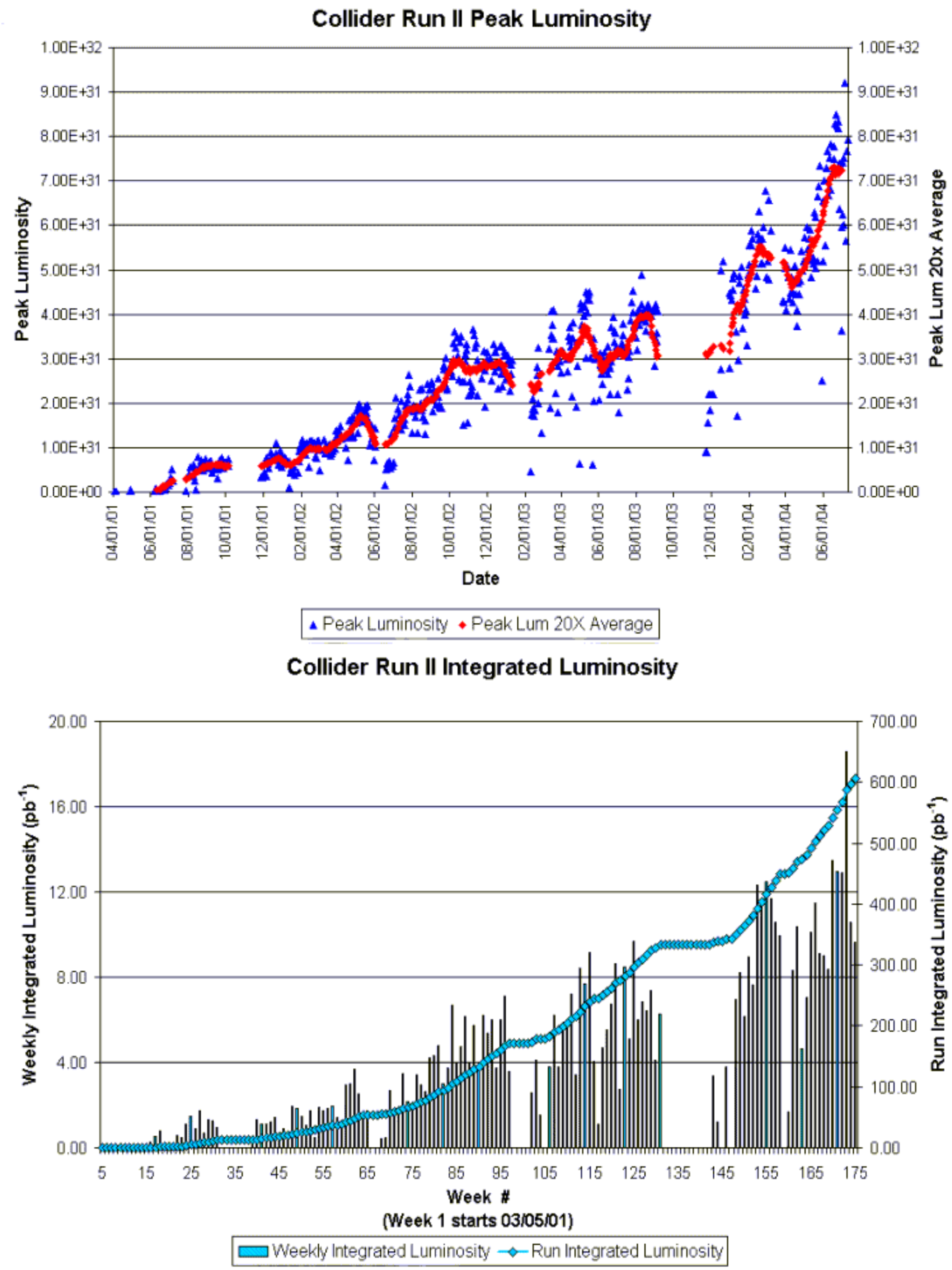
Seminar in CERN

July 13, 2004

Introduction

- We encountered multiple problems at the beginning of Run II (2001-2002). That forced us to reconsider both the strategy and the luminosity goal
- The following major items have been reassessed
  - Pbar product. and cooling scenario
  - Scenario of collider operation
  - Recycler operating scenario
- As result all three scenarios were significantly modified
  - The model of luminosity evolution played important role to set future Tevatron operating scenario
    - It also helps to understand our present problems



Peak and integrated Run II luminosities

- Major changes in Tevatron operating scenario
  - The luminosity goal was reduced from to $5\cdot10^{32}$ to $3\cdot10^{32}$ $cm^{-2}s^{-1}$
  - Total integrated luminosity goal was reduced from 15 fb^{-1} to 8 fb^{-1}
  - Antiproton recycling was canceled
    - Accurate consideration yielded that the gain in number of pbars due to recycling was illusive. Having longer stores allows one to stack the same number of pbars as with recycling and have the same luminosity integral
  - Collider operation with large number of bunches (140 x 121) was canceled and present 36 x 36 bunch operation will be used to the end of Run II
    - After more careful consideration both detectors increased the threshold on number on events per crossing. That combined with reduced luminosity goal allowed us to stay with present number of bunches without sacrificing luminosity integrated by detectors
    - This decision solves the problem with beam-beam effects due to parasitic collisions but puts severe limitation on total longitudinal emittance of antiprotons (3.3 times).
    - We are doing quite well now with longitudinal phase space in Accumulator but present Recycler has factor of 2 smaller phase space density
    - Future Recycler with electron cooling should solve this problem

Parametric Model of Luminosity Evolution

Takes into account the major beam heating and particle loss mechanisms

- **Phenomena taken into account**
  - ⇒ Interaction with residual gas
    - ♦ Emit. growth and particle loss due to E-M and nuclear scattering
  - ⇒ Particle interaction in IPs (proportional to the luminosity)
    - ♦ Emit. growth and particle loss due to E-M and nuclear scattering
  - ⇒ IBS
    - ♦ Energy spread and emittance growth due to multiple scattering
  - ⇒ Longitudinal dynamics
    - ♦ Nonlinearity and finite size of potential well
    - ♦ Bunch lengthening due to RF noise and IBS
    - ♦ Particle loss from the bucket due to single IBS (Touschek effect) and due to heating longitudinal degree of freedom (multiple IBS and RF noise)
    - ♦ Absence of tails after acceleration
- **Phenomena ignored in the model**
  - ⇒ Beam-beam effects
  - ⇒ Non-linearity of the lattice
  - ⇒ Diffusion amplification by coherent effects

Thus, it can be considered as **the best-case scenario.** It describes sufficiently well most stores

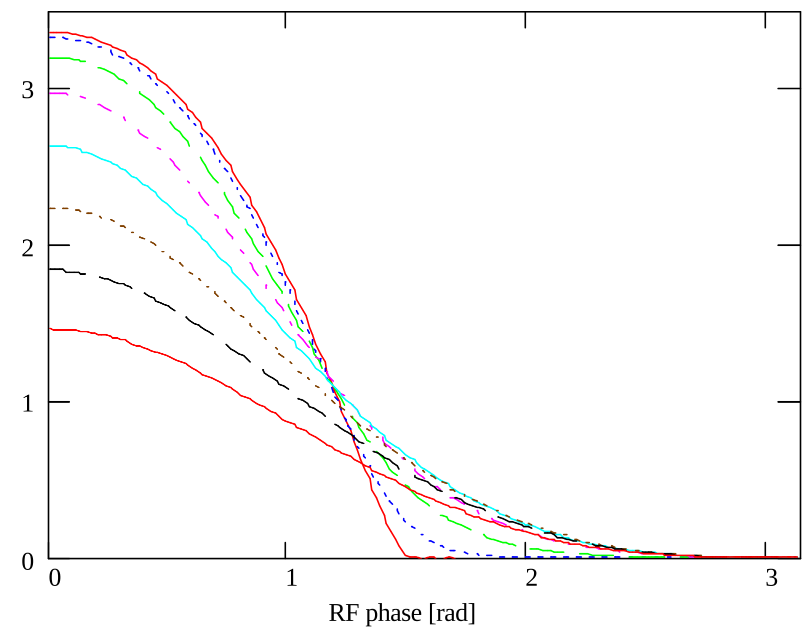
Beam Evolution in Longitudinal Degree of Freedom

- Longitudinal acceptance grows from 4 to 10 eV s during acceleration
  - Absence of tails after acceleration
- Interplay of single and multiple scattering
- The model based on a solution of integro-differential equation which describes both single and multiple IBS

$$\frac{\partial f}{\partial t} = \int_0^\infty W(I, I')\left(f(I', t) - f(I, t)\right) \mathrm{d}I'$$

where the kernel is

$$\tilde{W}(E, E') = \frac{D w_0 w w'}{L_C (E - E')^2} \begin{cases} \dfrac{1}{2w} + \dfrac{I}{E' - E} & , \quad E' \ge E + dE \,, \\ \dfrac{1}{2w'} + \dfrac{I'}{E - E'} & , \quad E' \le E - dE \,, \end{cases}$$



Numerical simulation of the longitudinal bunch profile evolution

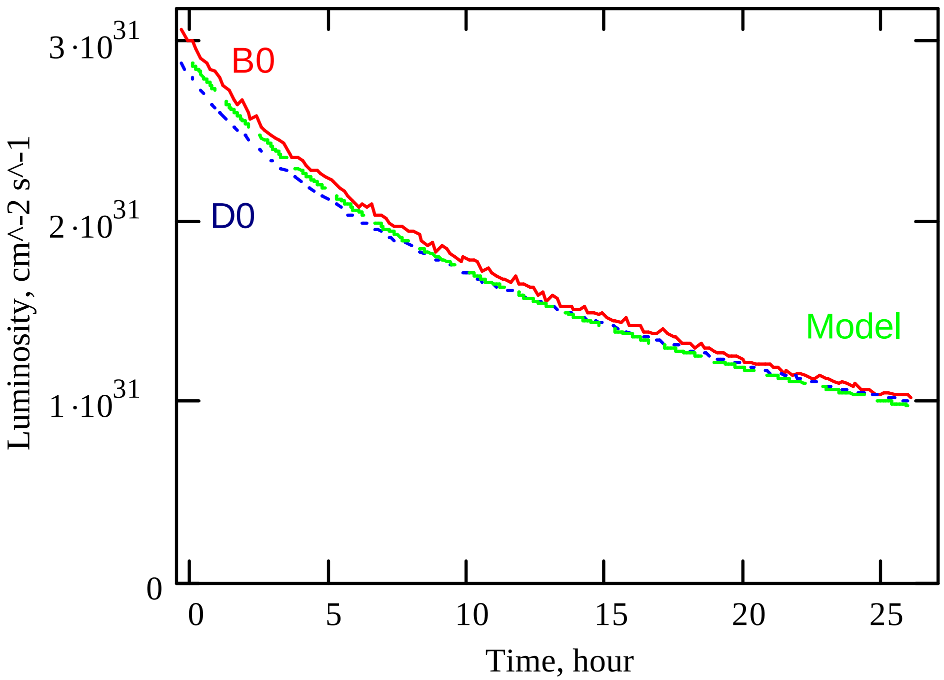
- Dependence of bunch length on time was parameterized as a simple expression of the bunch parameters

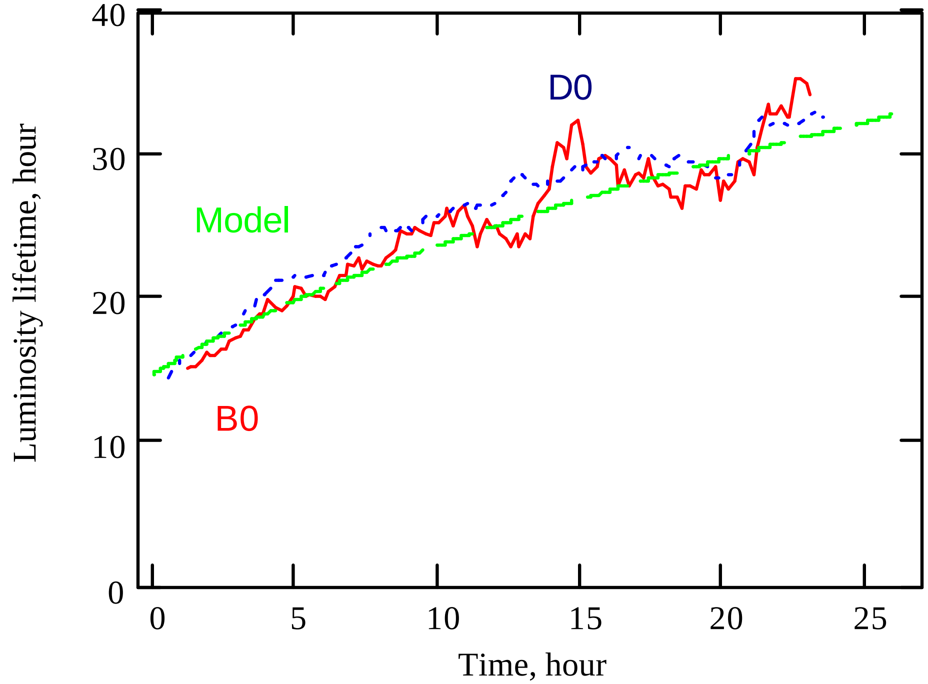
Parametric model of luminosity evolution

- Compromise between simplicity of the model and accuracy of the description
  - Finite accuracy of the measurements
  - System of eight ordinary differential equations

$$
\frac{d}{dt}\begin{bmatrix} e_{px} \\ e_{py} \\ s_{pp}^{\;2} \\ N_p \\ e_{ax} \\ e_{ay} \\ s_{pa}^{\;2} \\ N_a \end{bmatrix} = \begin{bmatrix} 2de_{px}/dt\big|_{BB} + de_{px}/dt\big|_{IBS} + de_{px}/dt\big|_{gas} \\ 2de_{py}/dt\big|_{BB} + de_{py}/dt\big|_{IBS} + de_{py}/dt\big|_{gas} \\ ds_{pp}^{\;2}/dt\big|_{total} \\ -N_p t_{scat}^{\;-1} - dN_p/dt\big|_L - 2Ls_{p\bar{p}}/n_b \\ 2de_{ax}/dt\big|_{BB} + de_{ax}/dt\big|_{IBS} + de_{ax}/dt\big|_{gas} \\ 2de_{ay}/dt\big|_{BB} + de_{ay}/dt\big|_{IBS} + de_{ay}/dt\big|_{gas} \\ ds_{pa}^{\;2}/dt\big|_{total} \\ -N_a t_{scat}^{\;-1} - dN_a/dt\big|_L - 2Ls_{p\bar{p}}/n_b \end{bmatrix}
$$

Luminosity Evolution

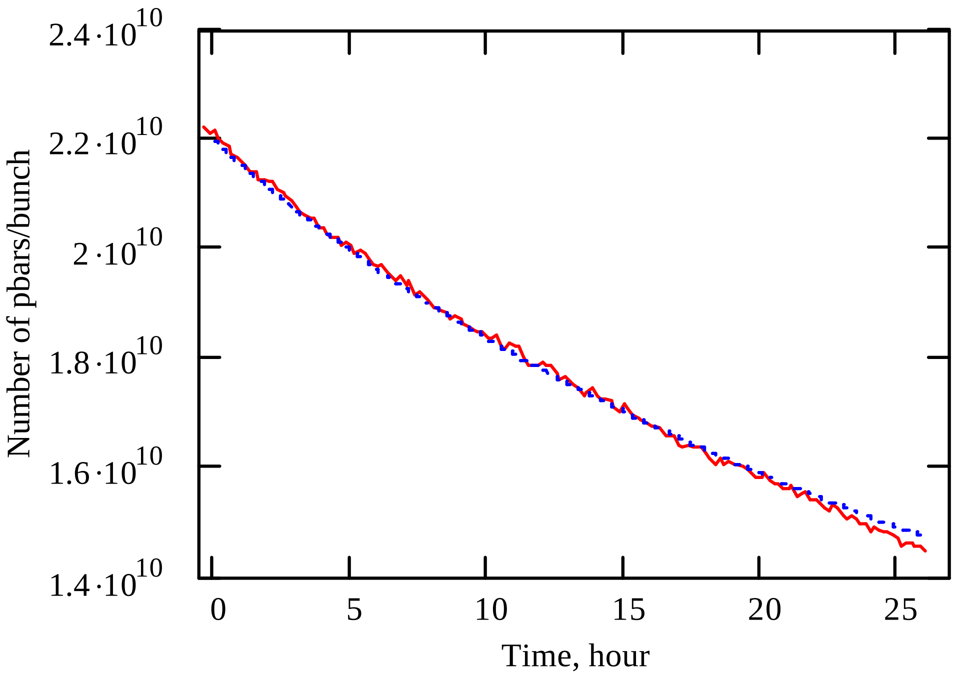


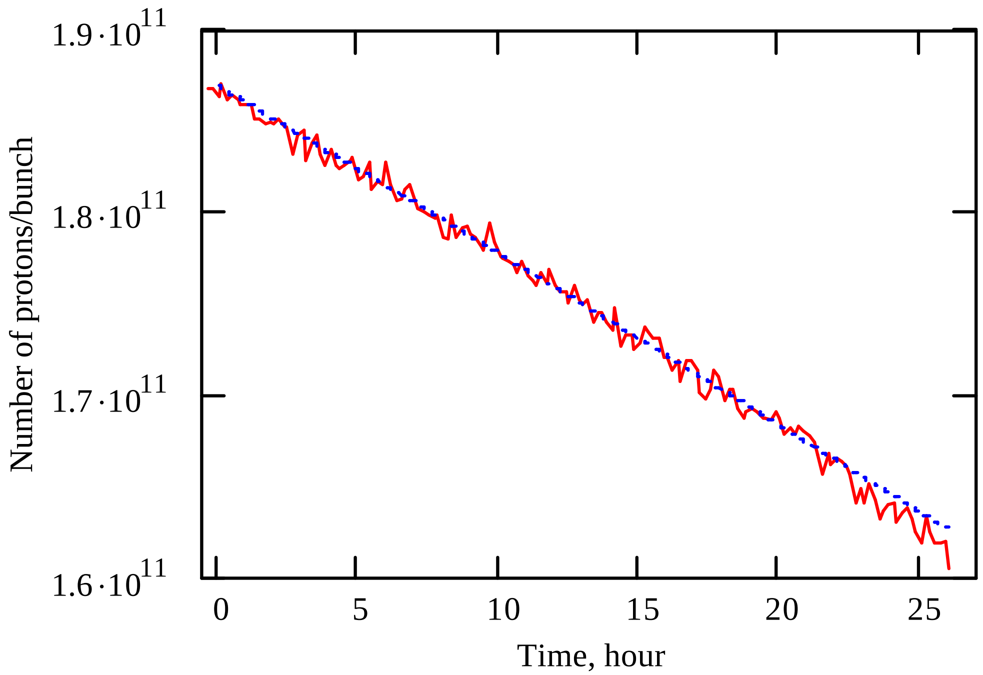


Good regular store !!! (Store 2138, Jan.5 2003)

- Three free parameters are used in the model
  - Residual gas pressure - $P=1\cdot10^{-9}$ Torr of N_2 equivalent
  - Spectral density of RF noise- $P_{ff}\left(f_s\right)\approx 5\cdot10^{-11}\quad \mathrm{rad}^2/\mathrm{Hz}$ (70μrad in Δf=100Hz)
  - X-Y coupling - k = 0.4 (strong coupling due to beam-beam effects)
  - Their values are not very critical for the luminosity prediction but important for detailed comparison

Number of particles per bunch (Store 2138)

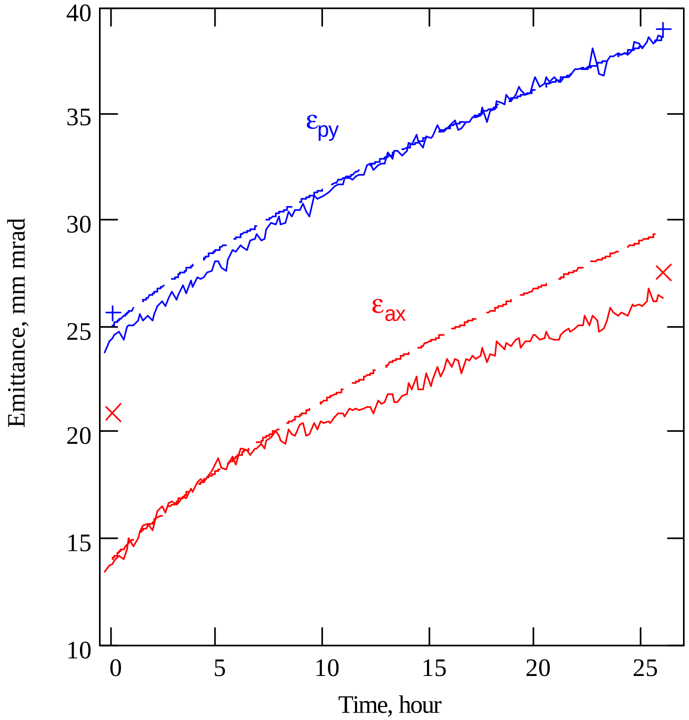


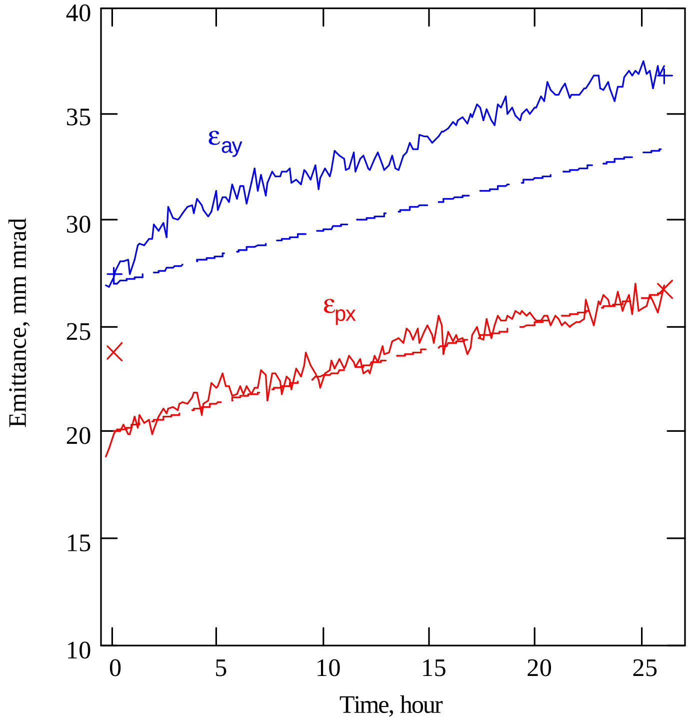


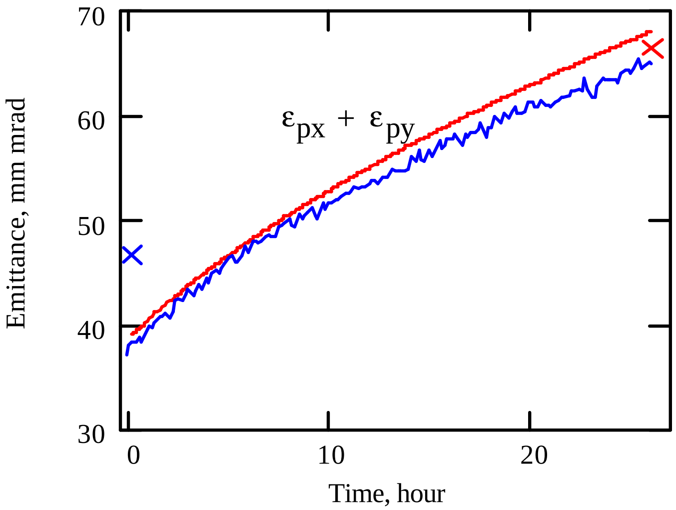
Pbars | Protons

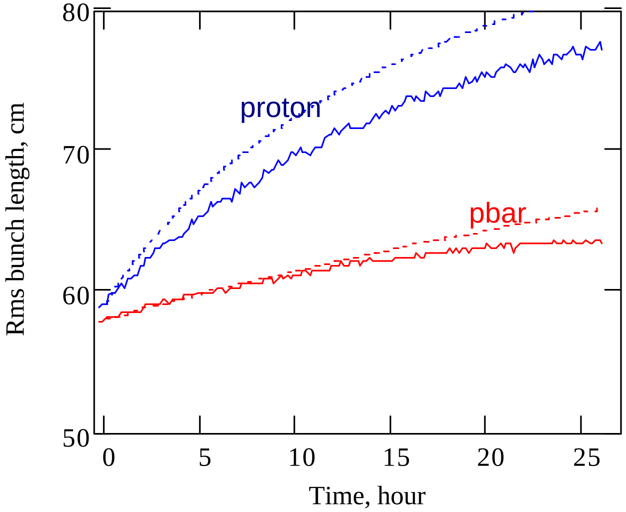
- **At the store beginning**
  - Pbar loss is large due to large initial luminosity
  - Proton loss is small due to short bunch length/absence of tails and, consequently, low longitudinal loss
  - At the store end the particle loss is accelerated because the beam approaches the scrapers
- **Model describes well the particle loss during the store**

Dependence of Emittances and bunch lengths on time for Store 2138



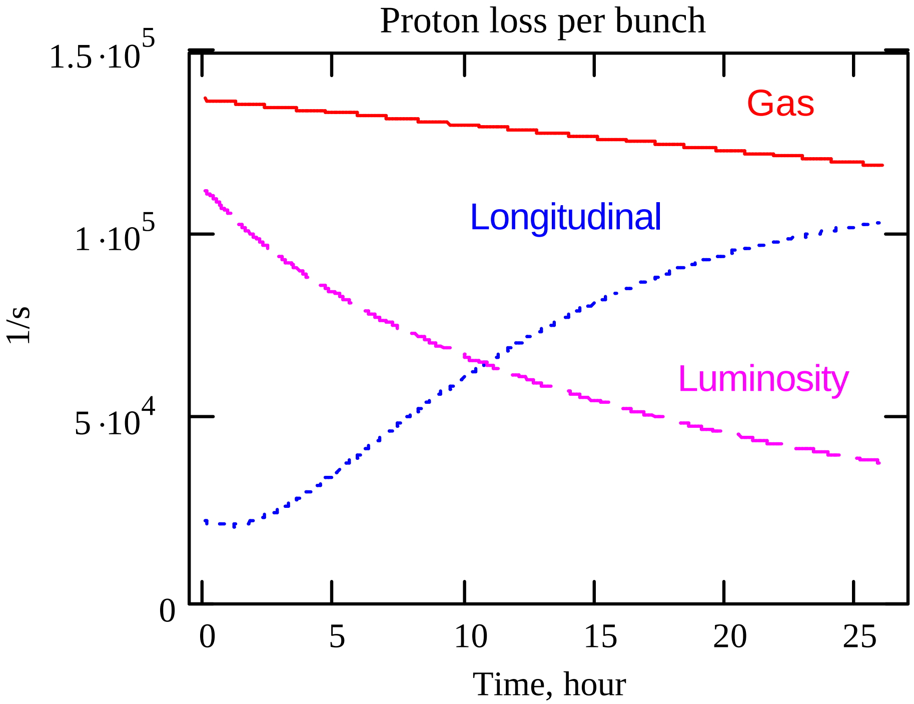


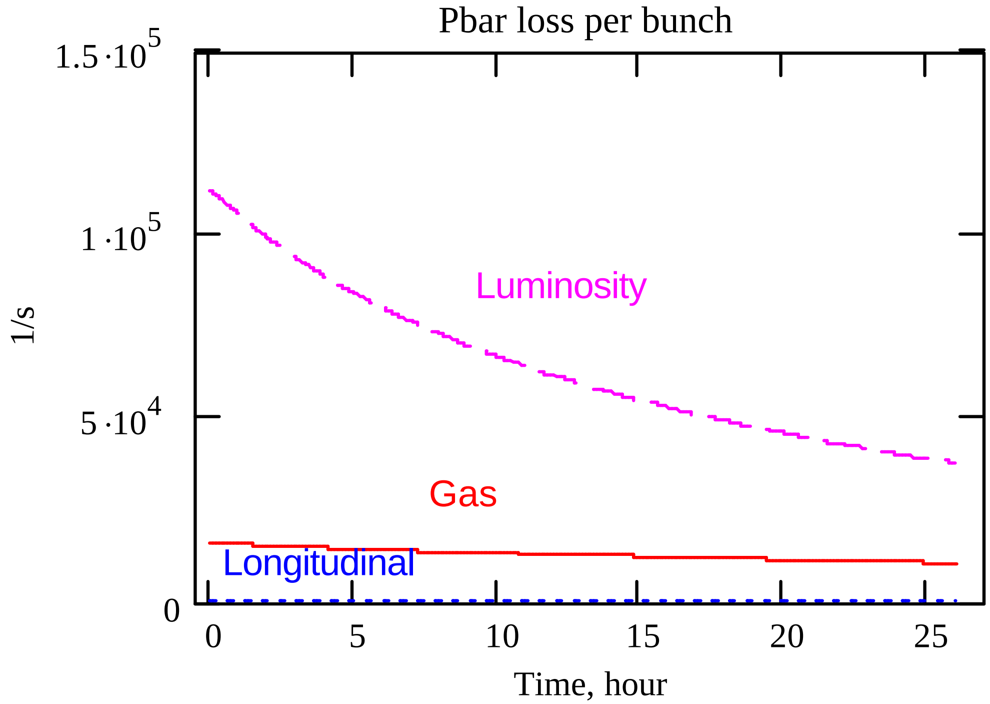




- **Beam-beam effects in pbar beam**
  - Vertical pbar emit. grows faster at the store beginning
  - Pbar bunch length does not grow at the second half of the store
- **Beam-beam effects in proton beam**
  - Coupling amplifies energy transfer from hor. to vert. plane at the store beginning (κ=0.4)

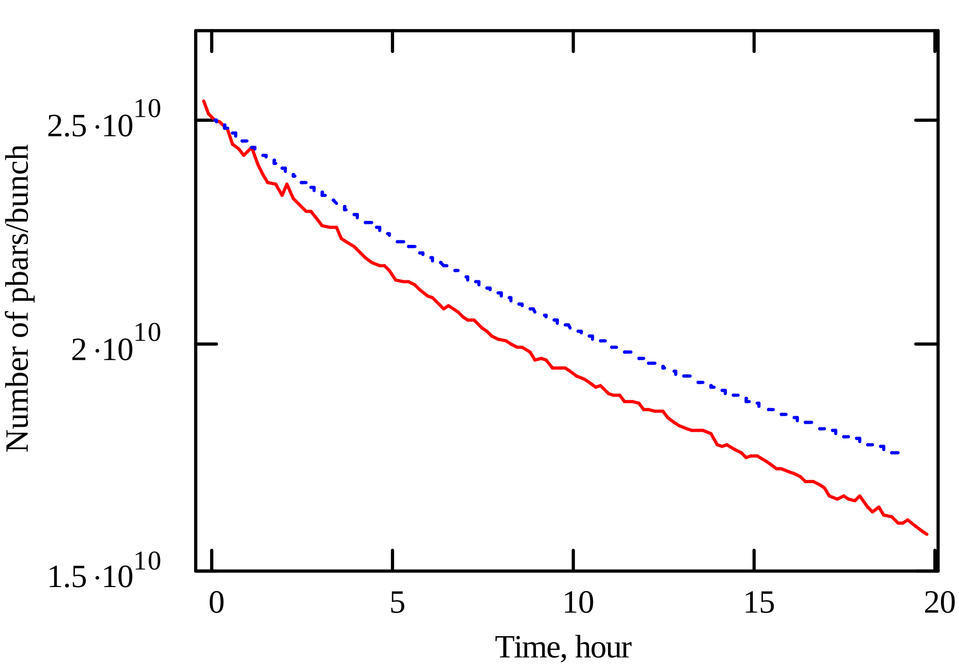
Particle loss computed for different loss mechanisms for Store 2138.

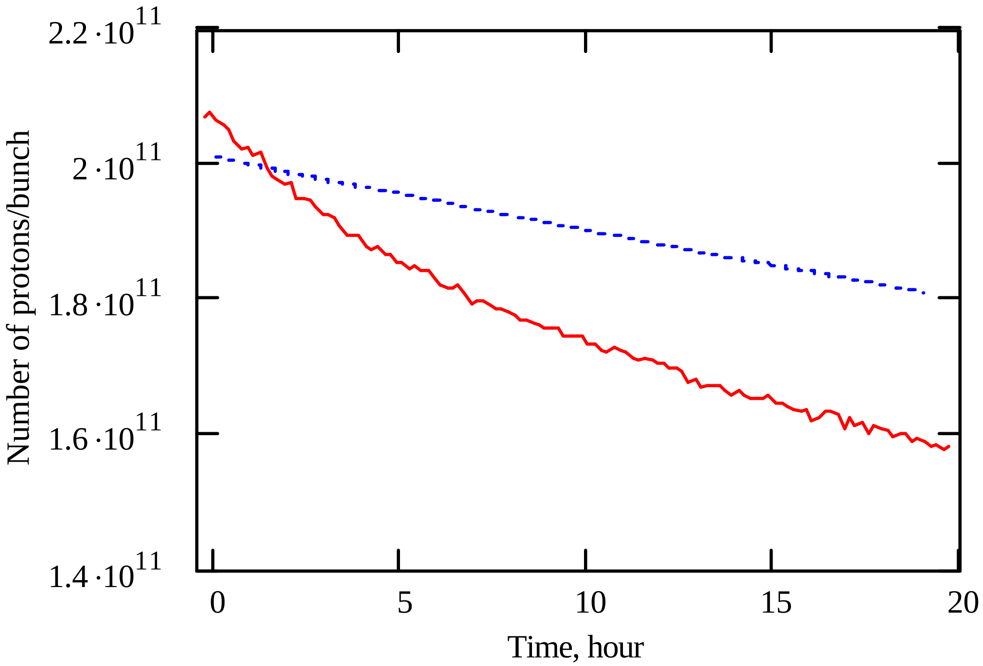




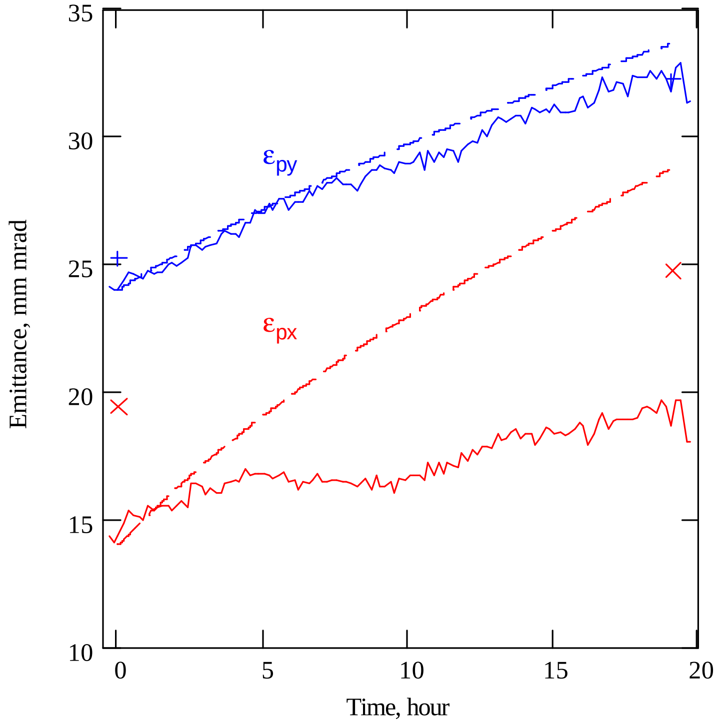
Comparison of Model Predictions to Store 2328 (Mar. 20 2003)

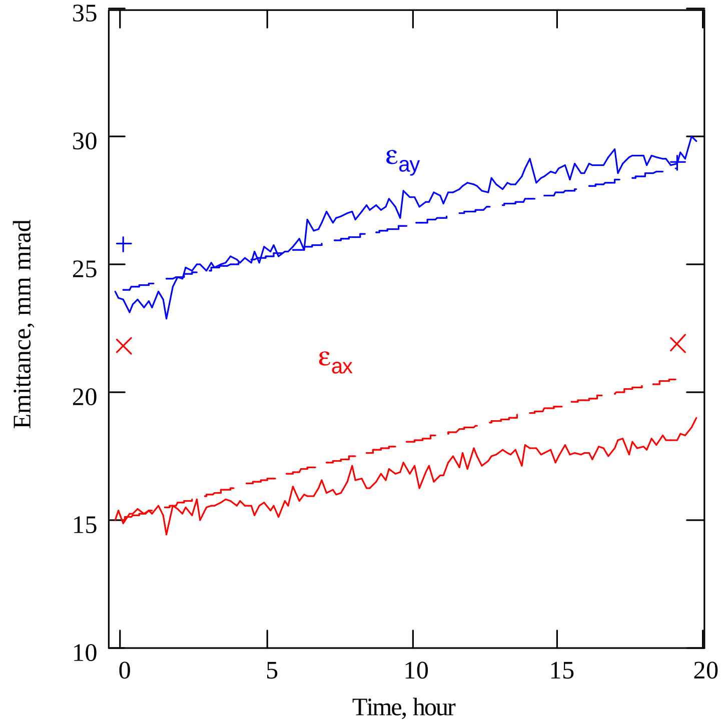
The store is strongly affected by the beam-beam effects !!!

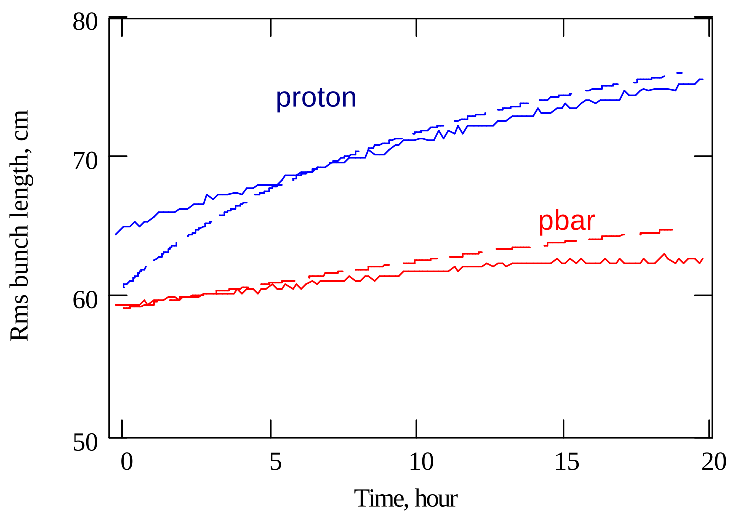




- At the store beginning both proton and pbar bunch intensities decay faster than the model predictions
  - Incorrect tune is the most probable reason

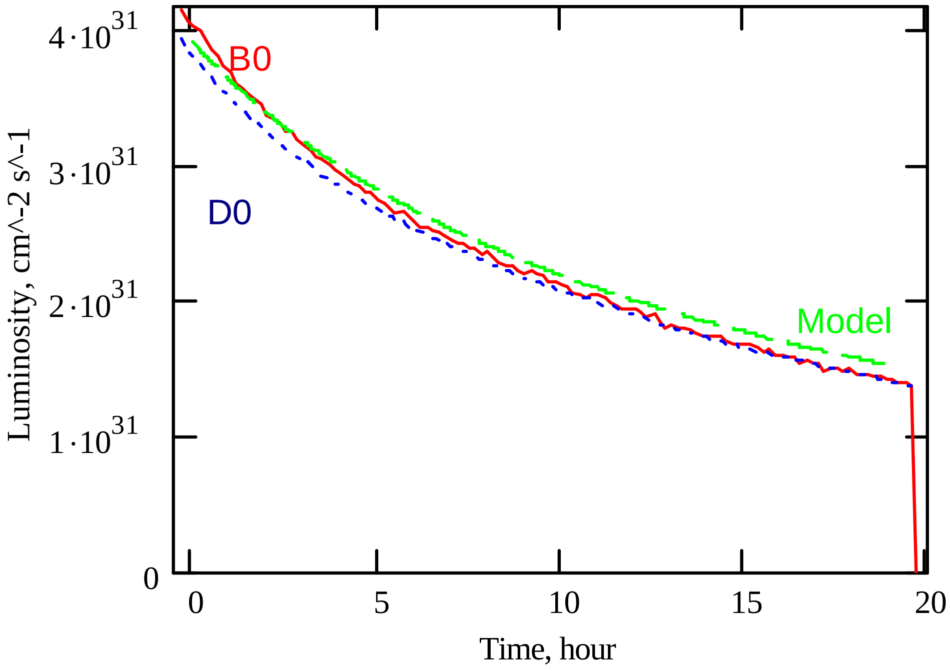


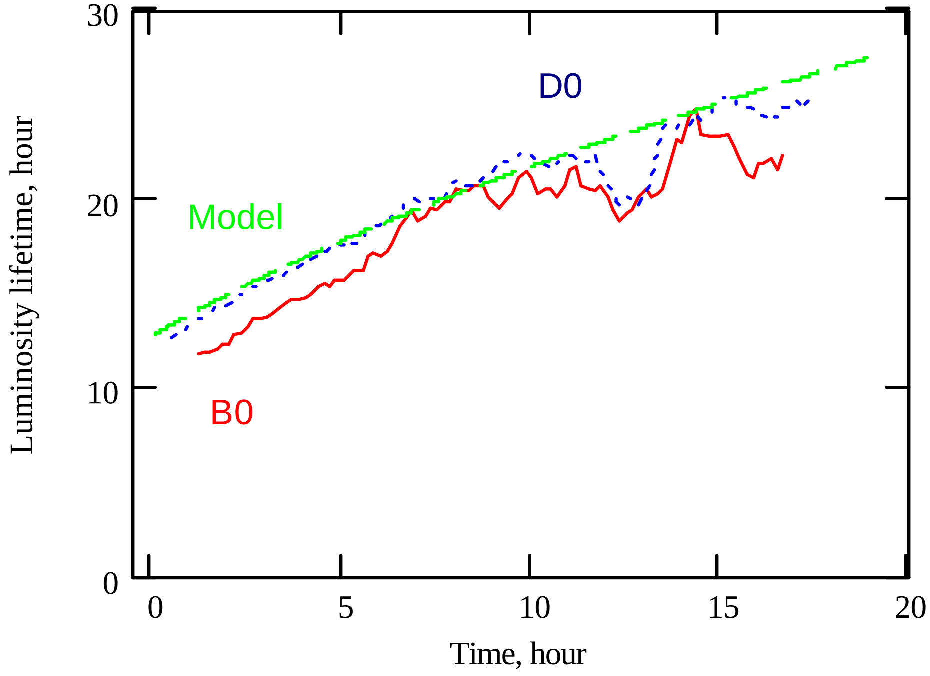




Emittances and bunch lengths on time for Store 2328

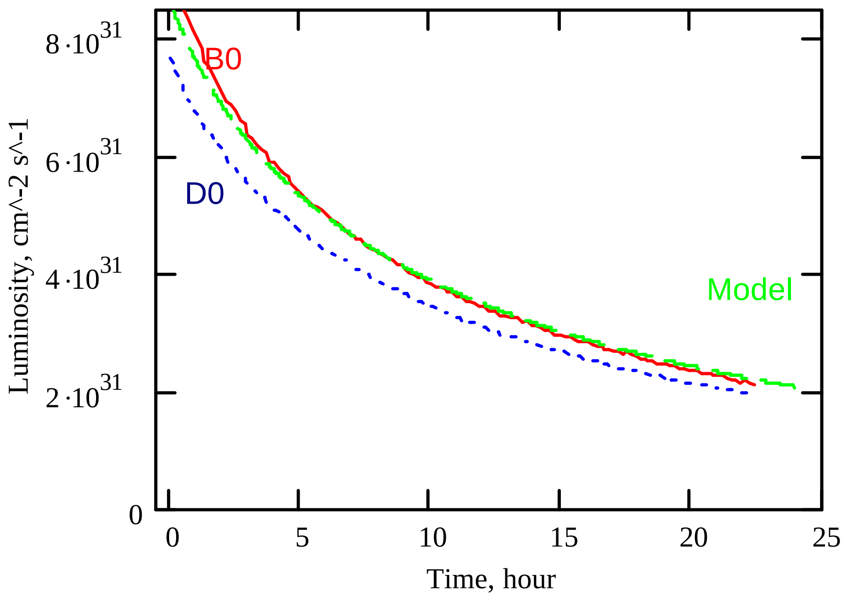
- **Beam-beam effects**
  - Proton bunch length and hor. emit. grow slower at the store beginning
    - Protons with large synchrotron amplitudes are lost due to beam-beam effects



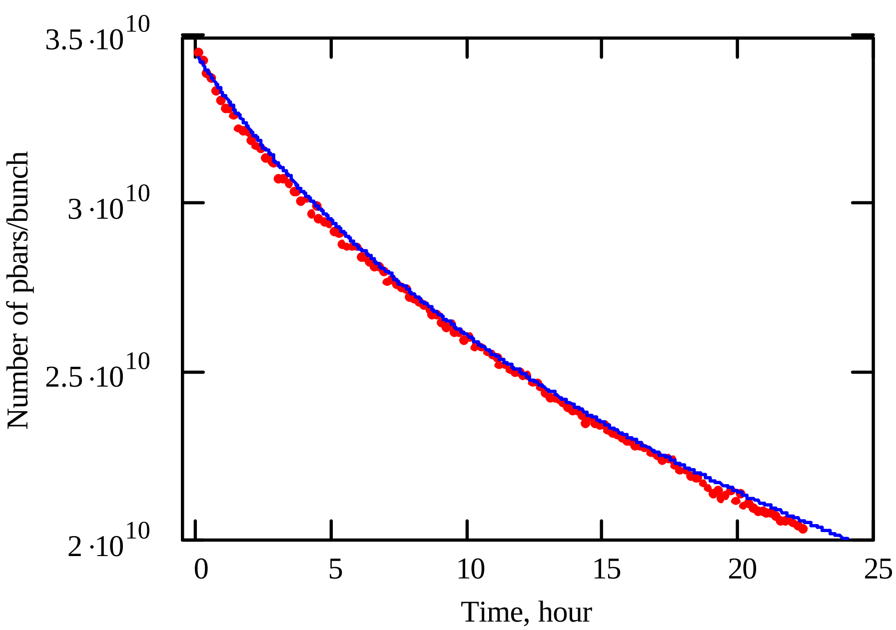


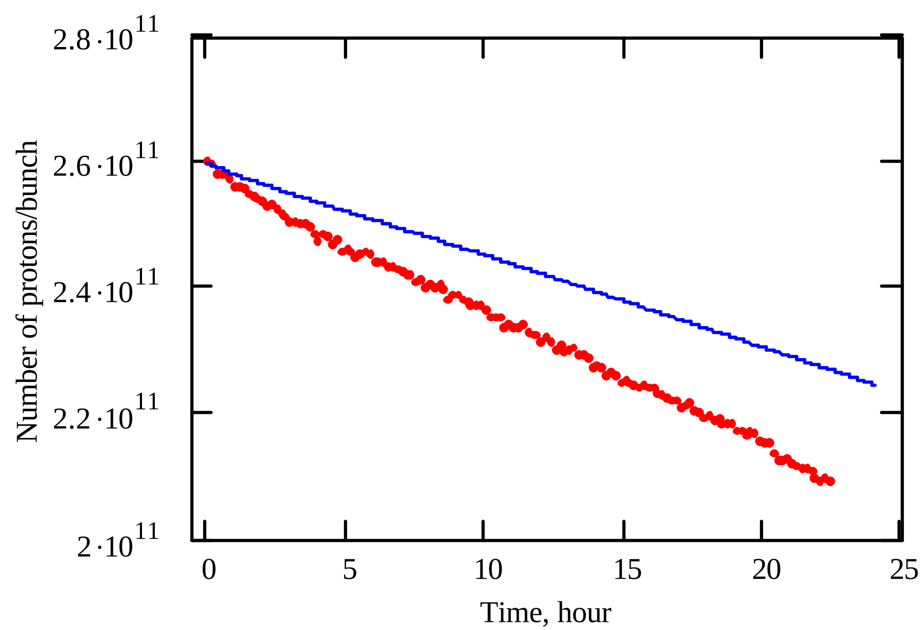
- Luminosity decays faster than the model predictions but the difference is small

Present best store (store 3588, June 22, 2004)

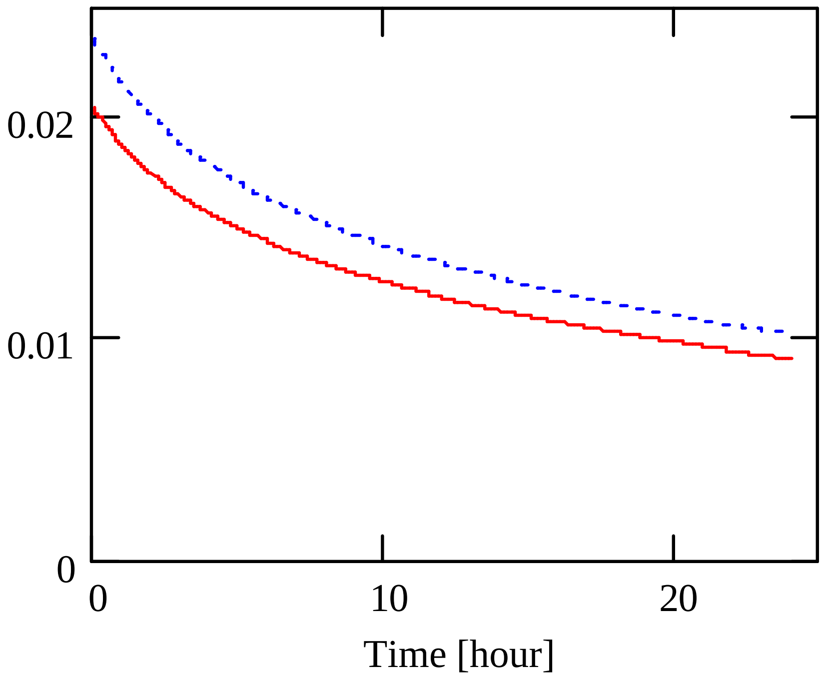


- Tevatron Run IIA is commissioned
- Further luminosity growth is mainly related with increase in Pbar production
- There are significant discrepancies between simple model and measurements due to beam-beam effects
- More accurate diffusion model should point out details how beam-beam effects work

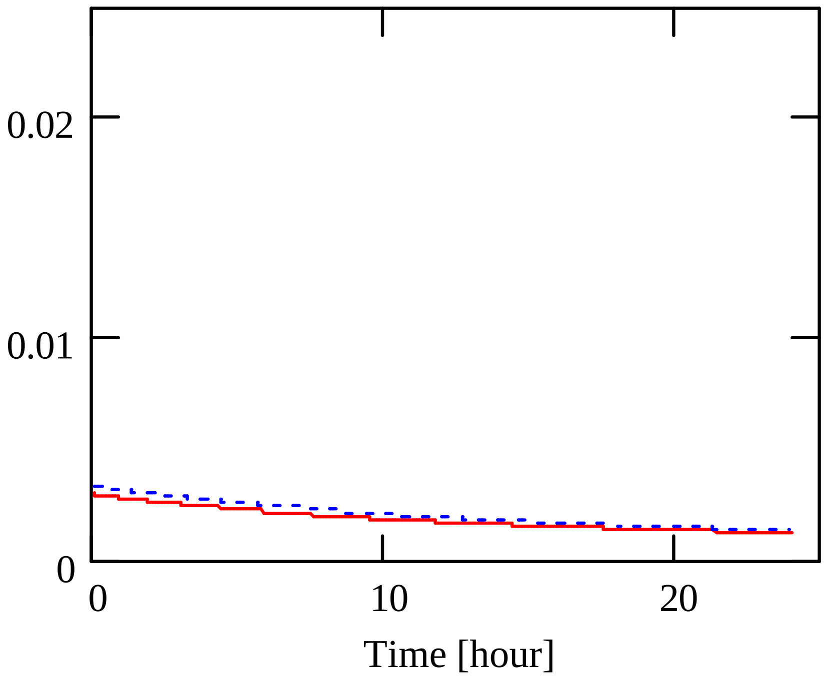




Computed head-on beam-beam linear tune shifts for Store 3588 (June. 22 2004)



Pbars



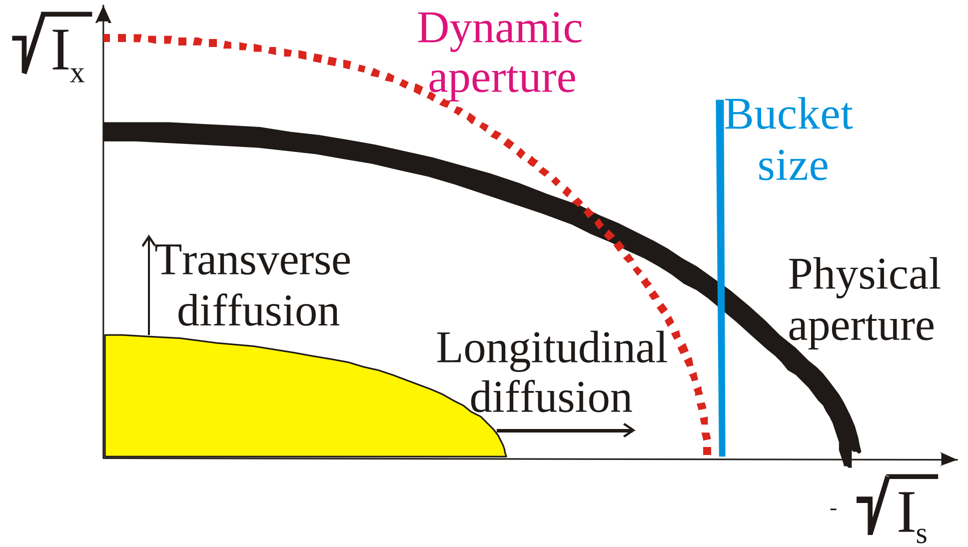
Protons

- We already above the design linear tune shift of 0.02 for pbar beam
  - But at about 30% for protons

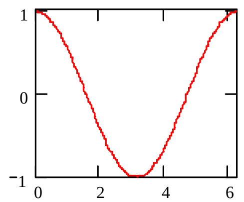
Particle Loss at Injection

- **Experimental observations before Tevatron realignment at the summer 2003 shutdown**
  - Proton lifetime at proton helix (1– 4 hour) is much worse than at central orbit (~10 hour)
  - Lifetime is affected by the machine chromaticity
    - Smaller chromaticity improves the lifetime but its reduction is limited by head-tail instability
  - Strong dependence of the lifetime on bunch length
  - Intensity lifetime is much worse than the emittance lifetimes
    - Proton intensity decays as $N\left(1-\sqrt{t/t}\right)$
  - Additionally to mentioned above, the **pbar lifetime** is strongly affected by beam-beam effects

♦ **Basic mechanisms and reasons of the proton loss**

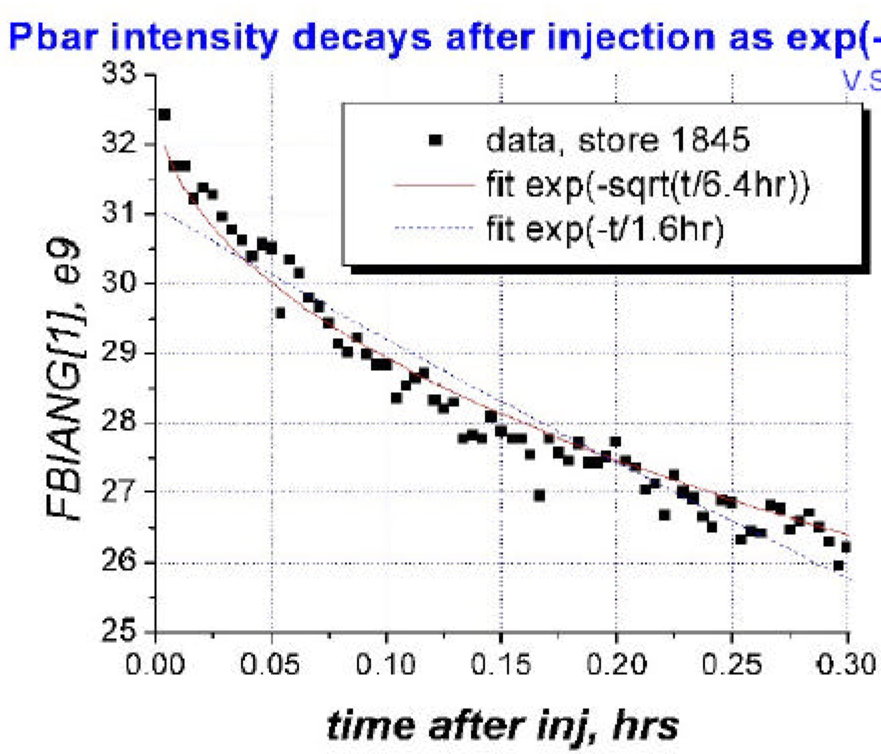


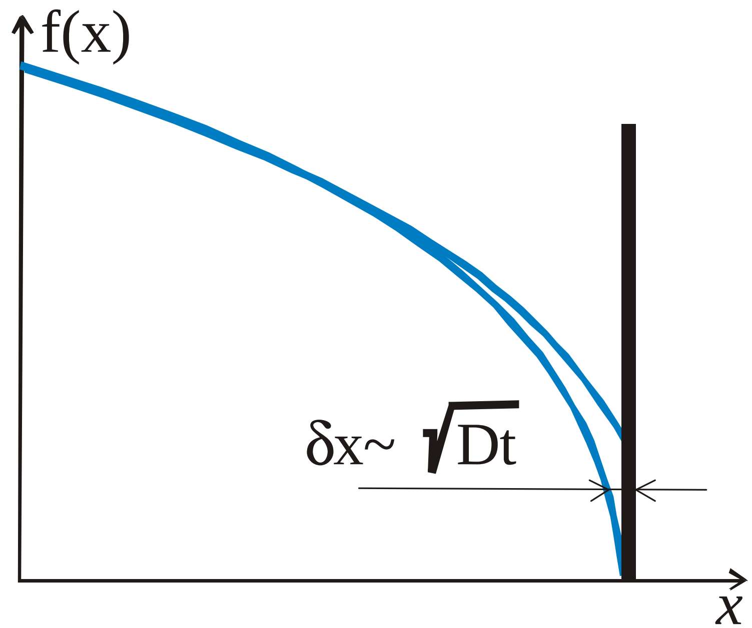
- Effects of longitudinal diffusion due to IBS and RF noise are amplified by
  - Overfilled bucket at injection
  - Shallowing the potential well near separatrix
  - Instability of motion at large synchrotron amplitudes



- Effects of transverse diffusion create loss due to
  - aperture limitations
  - reduced dynamic aperture for particles with large synchrotron apertures
- Major transverse diffusion mechanisms are
  - the residual gas scattering ($\left. d\varepsilon_x/dt\right|_{Gas} \approx \left. d\varepsilon_y/dt\right|_{Gas} \approx 1.1$ mm mrad/hour)
  - IBS (for protons $\left. d\varepsilon_x/dt\right|_{IBS} + \left. d\varepsilon_y/dt\right|_{IBS} \approx 1.2$ mm mrad/hour)

Measured dependence of proton beam intensity on time at injection



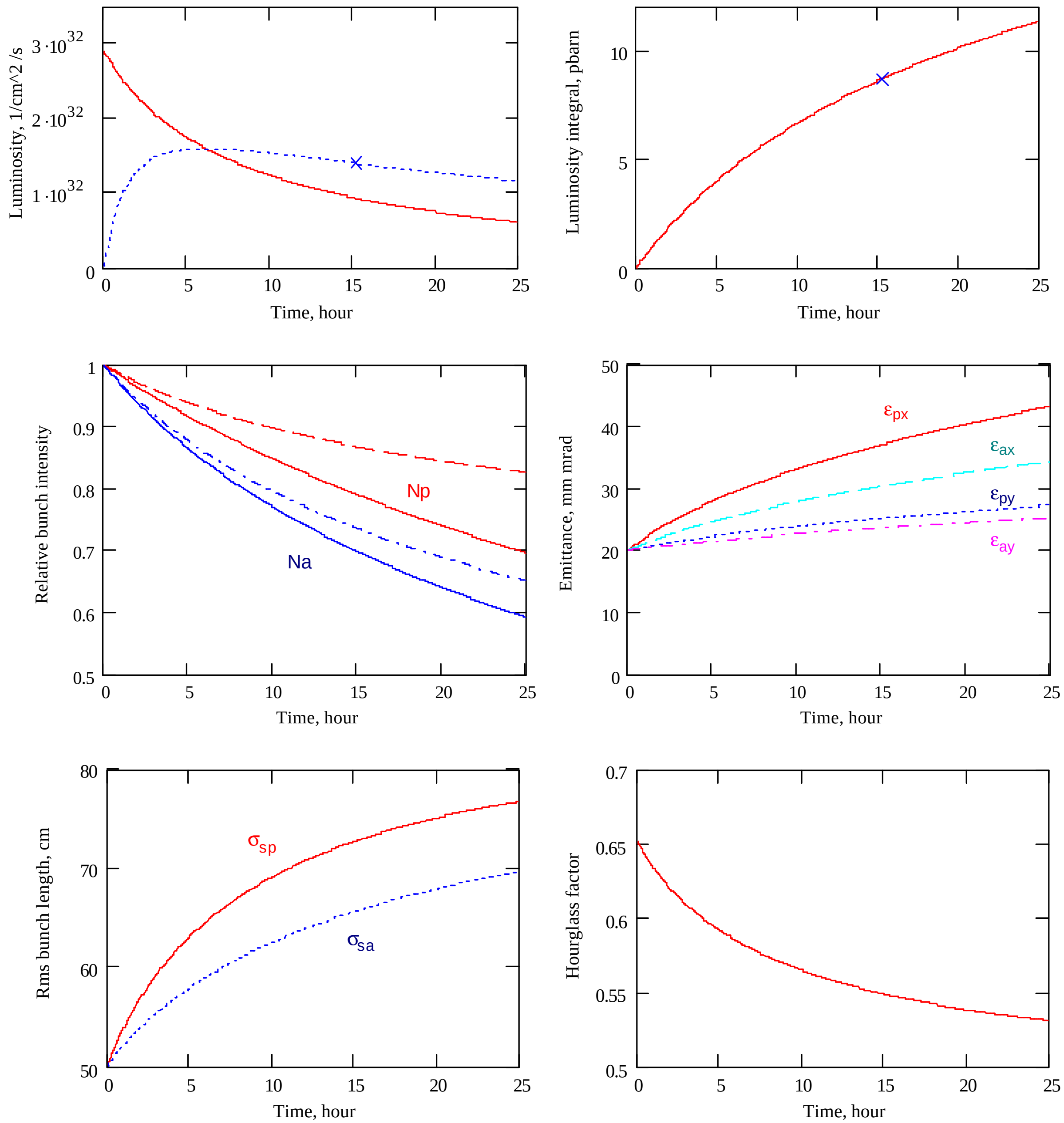


Improvements beam lifetime at injection after summer 2003 shutdown are related with

- Increased physical aperture
- Increased dynamic aperture
  - Better adjustments of feeddown sextupoles
  - Smaller chromaticity to suppress head-tail instability, reduced ⊥ impedance

Future Luminosity Scenario

- **Comparison of the model with present stores yields**
  - The model makes good prediction of luminosity lifetime even in the case when there is comparatively strong influence of beam-beam effects
    - We do not know all the details why we have “good” and “bad” stores
      - Incorrect tunes and chromaticity are the most probable reasons
      - New Shottky monitor (1.7 GHz) allows to track for every bunch
        - tunes (accuracy is compromised by large number of S-B lines)
        - chromaticities (different line widths for positive and negative freq.)
  - Many, but not all, stores have abnormal proton and pbar loss at the store beginning
- **That assures us that this simplified model can be used for prediction of luminosity integral for the final parameters of Run II**
  - This is the best case scenario
  - Beam-beam effects and instabilities needs to be addressed separately
  - Balanced approach for both Tevatron and Antiproton source parameters



Break-up of the collider luminosity lifetime

	Lifetime [hour]
Prot.intens.	52
Pbar.intens.	29
Prot.H.emit.	9
Prot.V.emit.	32
Pbar.H.emit.	17
Pbar.V.emit.	56
Hourglass factor	32
Luminosity	7.2

Present and final Run II parameters of the collider

	Store 2328 Mar 2003	Store 3588 June 2004	Final Run II
Number of protons per bunch, 10^{10}	20.7	26.0	27
Number of antiprotons per bunch, 10^{10}	2.54	3.44 (25%)	13.5
Norm. 95% proton emit., e_x / e_y, mm mrad	~14/24	~20/15	20/20
Norm. 95% pbar emit., e_x / e_y, mm mrad	~15/24	~18/14	20/20
Proton bunch length, cm	65	51	50
Antiproton bunch length, cm	59	49	50
Initial luminosity, 10^{30} cm^{-2}s^{-1}	40.5	84.5(29%)	290
Initial luminosity lifetime, hour	11	8.5	7.1
Store duration, hour	19	22.8	15.2
Luminosity integral per store, pbarn	1.71	3.1	8.65
Shot setup time, hour	2	2.5	2
Luminosity integral per year, fbarn^{-1}	-	0.30 (30%*)	2.4
Transfer efficiency: stack to Tev at low-beta	60%	68%	80%
Average pbar production rate, 10^{10}/hour	-	11 (27%)	40
Total antiproton stack size, 10^{10}	166	199 (32%)	610
Antiprotons extracted from the stack, 10^{10}	154	182(30%)	610

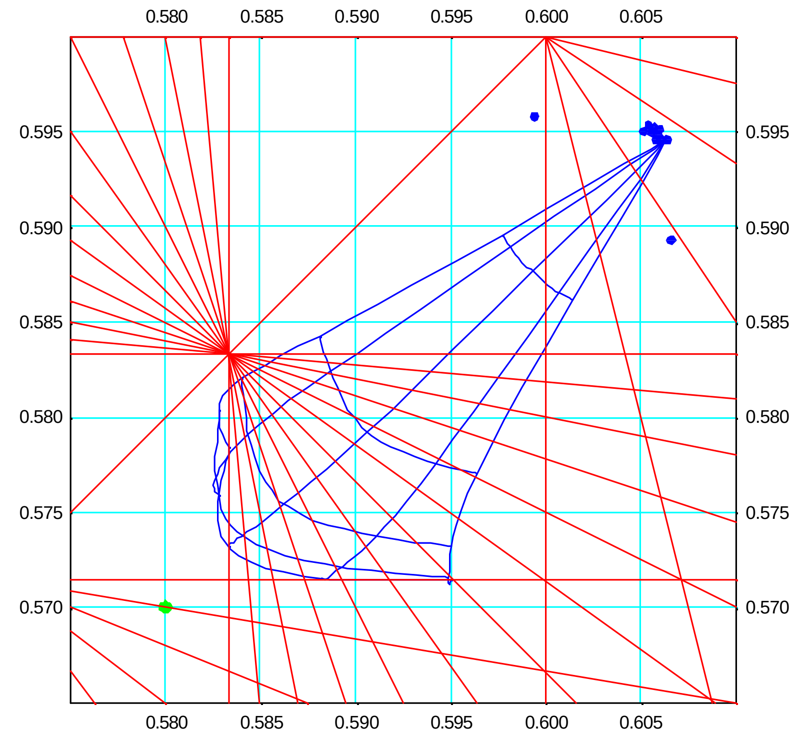
* Percentage is computed for our present best week (17 pbarn^{-1}) and 42 weeks/year of collider operation

Improvements of the Luminosity evolution model

- To improve the model we are carrying out the following actions
  - Theory
    - Simulation of the beam-beam effects in the presence of external noise
    - Putting more accurate integro-differential equations to describe evolution of transverse and longitudinal distributions
  - Experimental studies require a detailed knowledge of evolution of the transverse and longitudinal distribution functions and the tunes
    - That implies improvements in diagnostics
      - Tunes for each bunch are recorded to SDA (1.7 GHz Schottky)
        - Further improvements of tune measurements are required
        - “Another” Schottky monitor is launched
      - On-line measurements of longitudinal distribution function during a store is at last stages of development
        - Raw SBD data are in SDA
        - On-line reconstruction of longitudinal distribution will follow

Beam-Beam effects

- Tunes are between 5-th and 7-th, and on 12-th order resonance
  - 5-th and 7-th order are excited by long-range col. and lattice nonlinearity
  - 12-th order are excited by head-on
- Long range interactions make different tune shifts for different bunches
- Distance between 5-th and 7-th order resonances is 0.0285
  - Pbars from Protons
    - Head-on – 2·0.01=0.02
    - Long range within a bunch – 0.005
    - Bunch to bunch difference – 0.007
  - For the final Run II parameters protons experience only half of this tune shift due to smaller pbar intensity

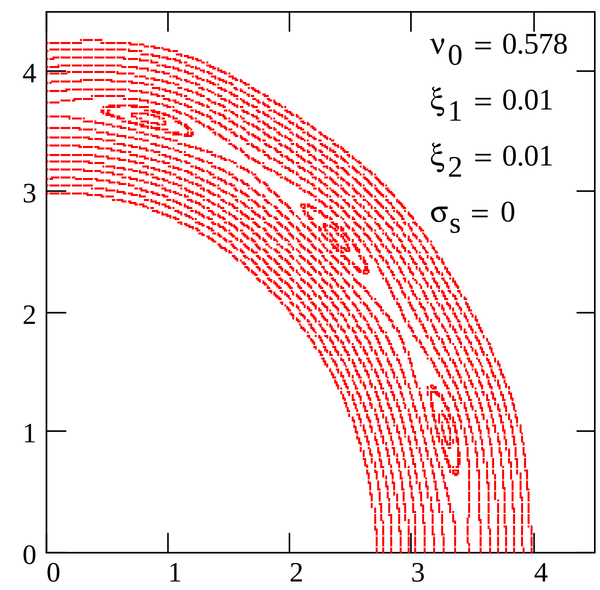


Footprint of pbar bunch #6 in the tune diagram with ν_x=0.580, ν_y=0.570 (green dot) and nominal beam parameters. Blue dots show small amplitude tunes for other bunches. Footprint lines go in 2σ and 22.5 deg in the space of actions (angle or $(a_x^2 + a_y^2)^{1/2}$ =const on a line).

Courtesy of Yu. Alexahin

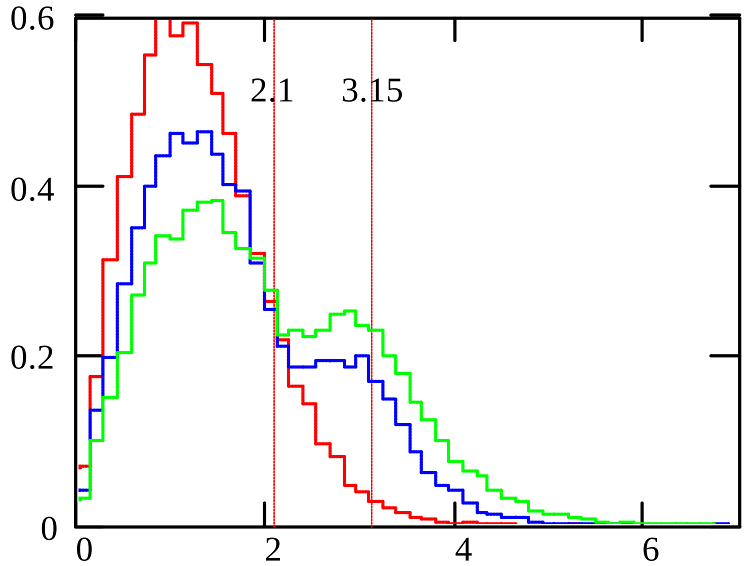
Approach to simulations

- 1 store ~ $4 \cdot 10^9$ turns – too much for any computer in visible future
- **Conclusion following from parametric model study**: for correctly tuned collider at present intensities the beam-beam effects and machine nonlinearity do not produce harmful effects on the beam dynamics while beams are in collisions
- We can not accept any significant worsening of the lifetime if we want to maximize the luminosity integral
- Beam-beam theory should be build as perturbation theory to the diffusion model
- Diffusion amplification by resonances
  - Motion inside resonance island is fast comparing to the beam lifetime
    - 100-10,000 turns depending on ξ and the resonance order

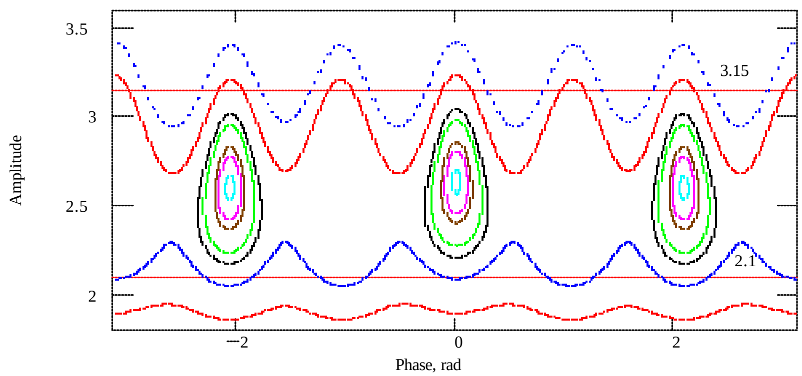


Phase trajectories in vicinity of 12-th order resonance ν_x=7/12; two Tevatron IPs but zero length of counter-rotating bunch, and zero synchrotron motion amplitude

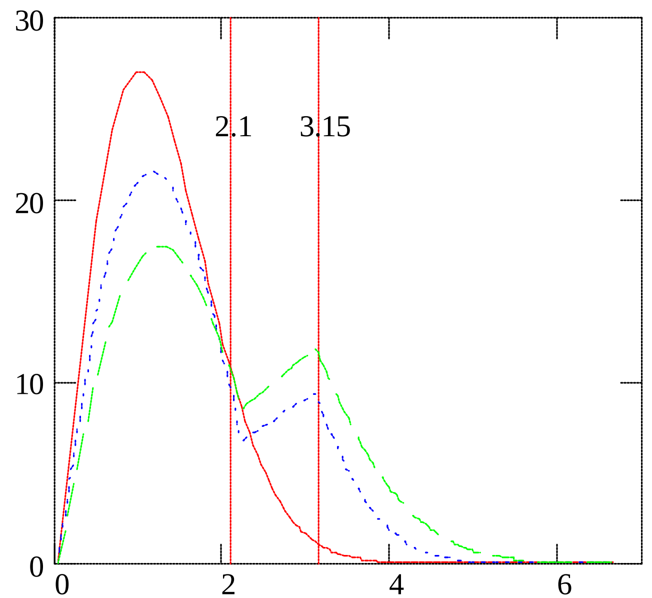
Flattening distribution over resonance



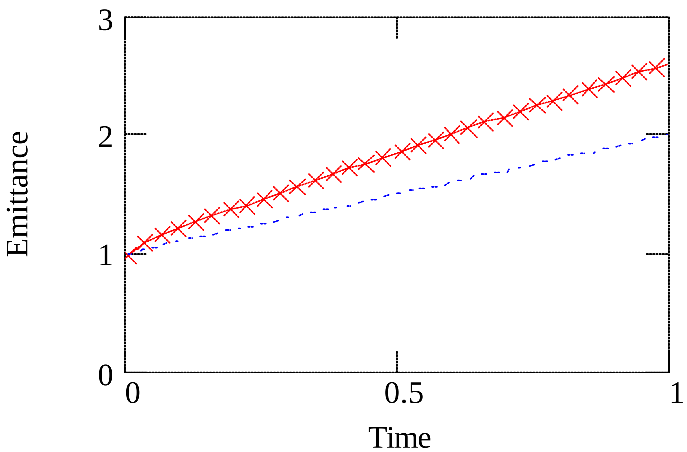
$\nu=0.325$, $\xi=0.02$, $\Delta p/p=0$, $s_s \ll b^*$



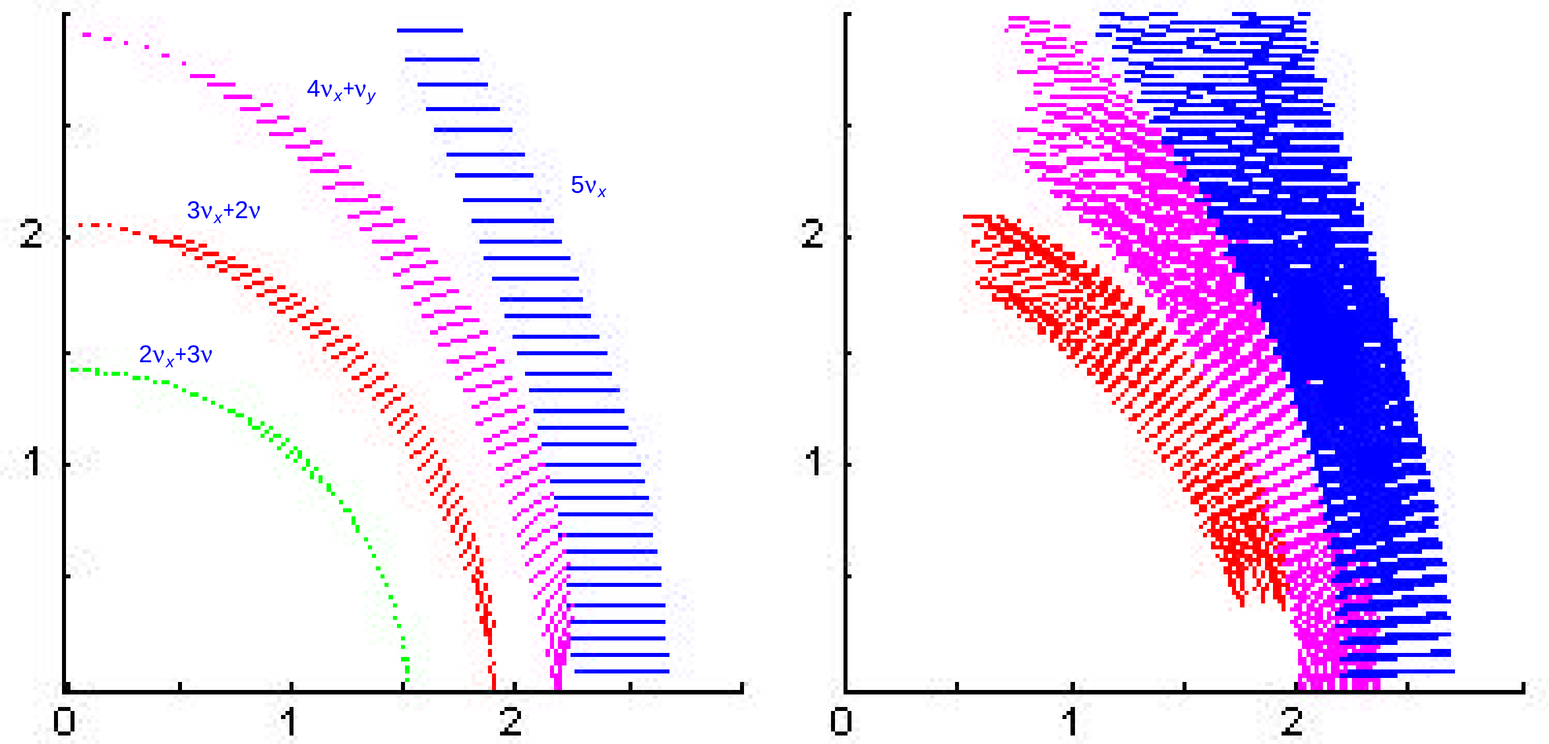
Tracking of 20,000 particles for 5,000 turns in vicinity of 6-th order resonance, $n_x = 2/6$



Particle distribution obtained by diffusion equation solving



Long Range collisions



d_p = 0 $\quad\quad$ $d_p = 1.25{\cdot}10^{-4}$

Swing of the normalized transverse amplitudes on the 5th order resonances and their synchrotron satellites at synchrotron amplitude d_p = 0 (left) and $d_p = 1.25{\cdot}10^{-4}$ (right), lattice chromaticity is zero, n_x = 20.585, n_y = 20.575.

Courtesy of Yu. Alexahin

How to perform tracking

- **The number of turns is determined by**
  - Decoherence of particle motion

$$\frac{\Delta a}{a} = \sqrt{Dn} \qquad \longrightarrow \qquad dm \approx 2p\, dQ\, n \approx 2p \frac{x}{10} \sqrt{Dn}\, n$$

$$dQ \approx \frac{x}{10} \frac{\Delta a}{a}$$

    - For $(\mathrm{Df}_0)^{-1}$ = 10 hour and ξ = 0.02 we obtain $\delta\mu = \pi/2$ for n ~30000 turns
    - For high order resonances decoherence occurs much faster $\propto 1/m$
  - Maximum acceptable noise
    - During one revolution around the resonance island the particle displacement due to diffusion has to be much smaller than the island width
      - Resonances with width above ~ 0.05σ need to be taken into account (5 such resonances change emittance growth time by ~25%)
      - For 12-th order resonances one revolution is about few thousand turns
      - For displacement of about 0.1 of resonance width we obtain the minimum number of turns

$$\Delta a \equiv \Delta \sqrt{N_{res}} << 0.05s \quad \xrightarrow{\Delta \approx 3\cdot 10^{-4}\ s} \quad N = \frac{\Delta e}{\Delta^2} \approx \frac{0.1}{\left(3\cdot 10^{-4}\right)^2} \approx \mathbf{10^6\ turns}$$

How to perform tracking (continue)

- Number of particles is determined by
  - The statistic accuracy of emittance calculations

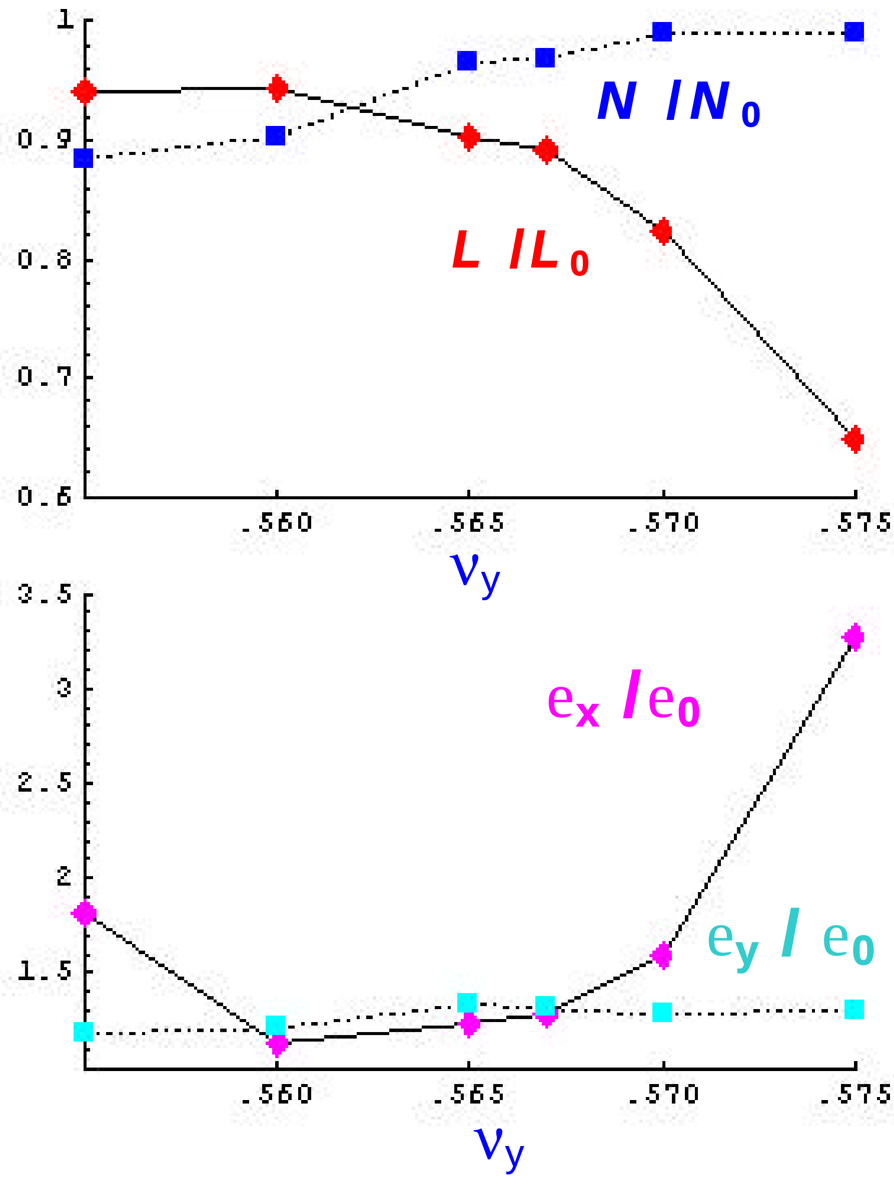
    1% accuracy of emittance calculations requires about 10,000 particles
  - Coverage of the phase space by particles for 3-D phase space

    10,000 particles correspond to an average particle distance (in 3 dimensional action phase space) of about 0.05σ

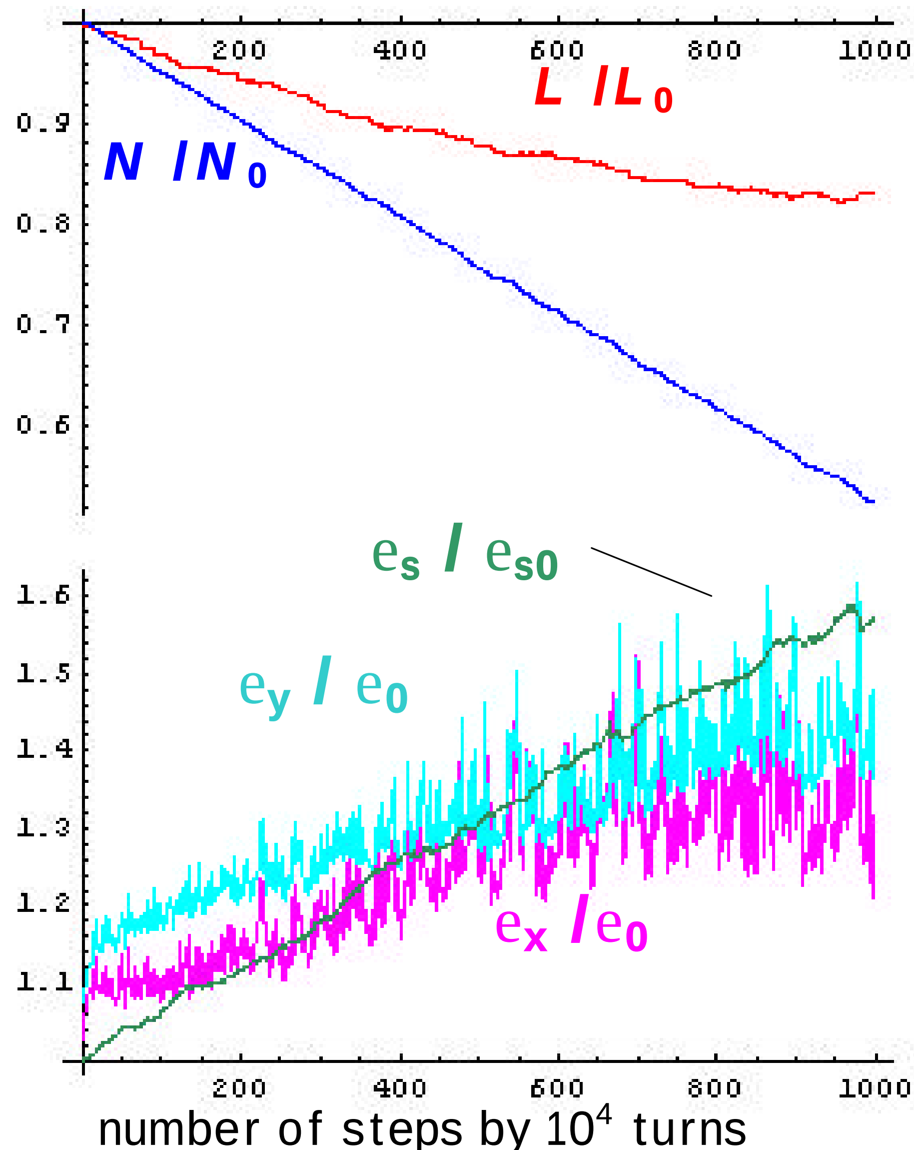
Beam-beam simulations with LifeTrack

- The model includes
  - Weak-strong interaction
  - Linear lattice with built-in chromatisities of tunes and beta-functions.
    - Beta-function chromaticities are excited by the final focus quadrupoles, $(b/p)\cdot(db/dp) \approx 500$.
    - X-Y coupling measured in Tevatron is included
  - Beam-beam kicks
    - Kicks are computed for a bunch with gaussian distribution. The bunch rolls due to coupling are taken into account
    - Single kick at parasitic collisions
    - Multiple kicks in main IPs to take into account the phase averaging
  - Synusoidal RF voltage
  - Diffusion is simulated by random kicks in all 3 degrees of freedom
  - Using different particles weight allows one to reduce the number of particles
- We are testing software to understand how to scale simulations to real Tevatron parameters
  - $\sim 2\cdot 10^6$ turns and 5,000 particles are required for reliable simulations

Beam-beam simulations with LifeTrack (continue)

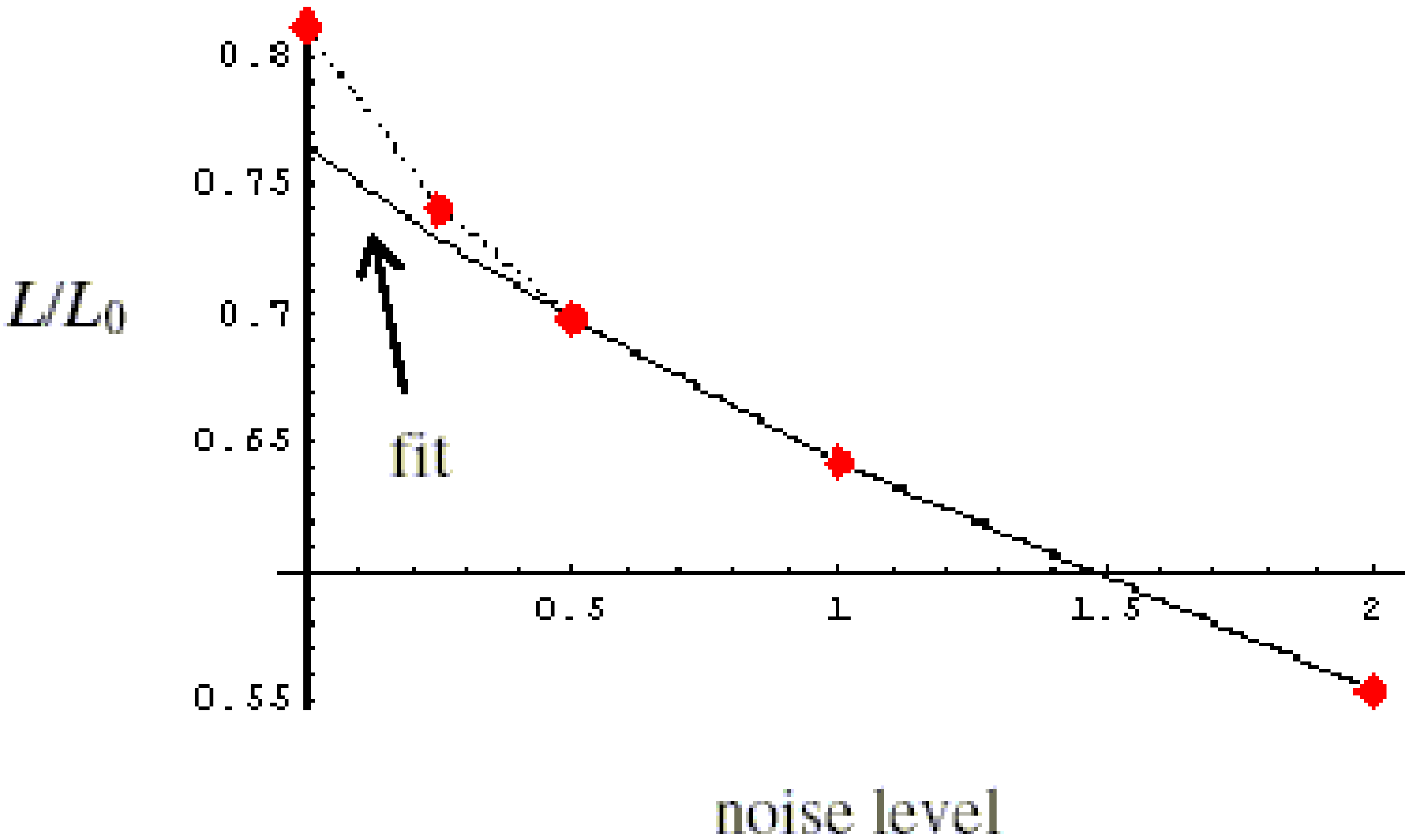


Specific luminosity for Tune scan along the main diagonal (lattice tunes $\nu_x = \nu_y + .01$), 2×10^6 turns



Tracking at WP $\nu_x = .57$, $\nu_y = .56$ for 10^7 turns, noise level corresponds to 20% emit. growth after 2×10^6 turns

Alexahin, Valishev, Shatilov



Dependence of luminosity on the noise level for 2×10^6 turns; the noise level equal to 1 corresponds to 20% emit. growth after 2×10^6 turns

Fit using three data points yields: $L/L_0 = 0.765/(1+0.19x)$

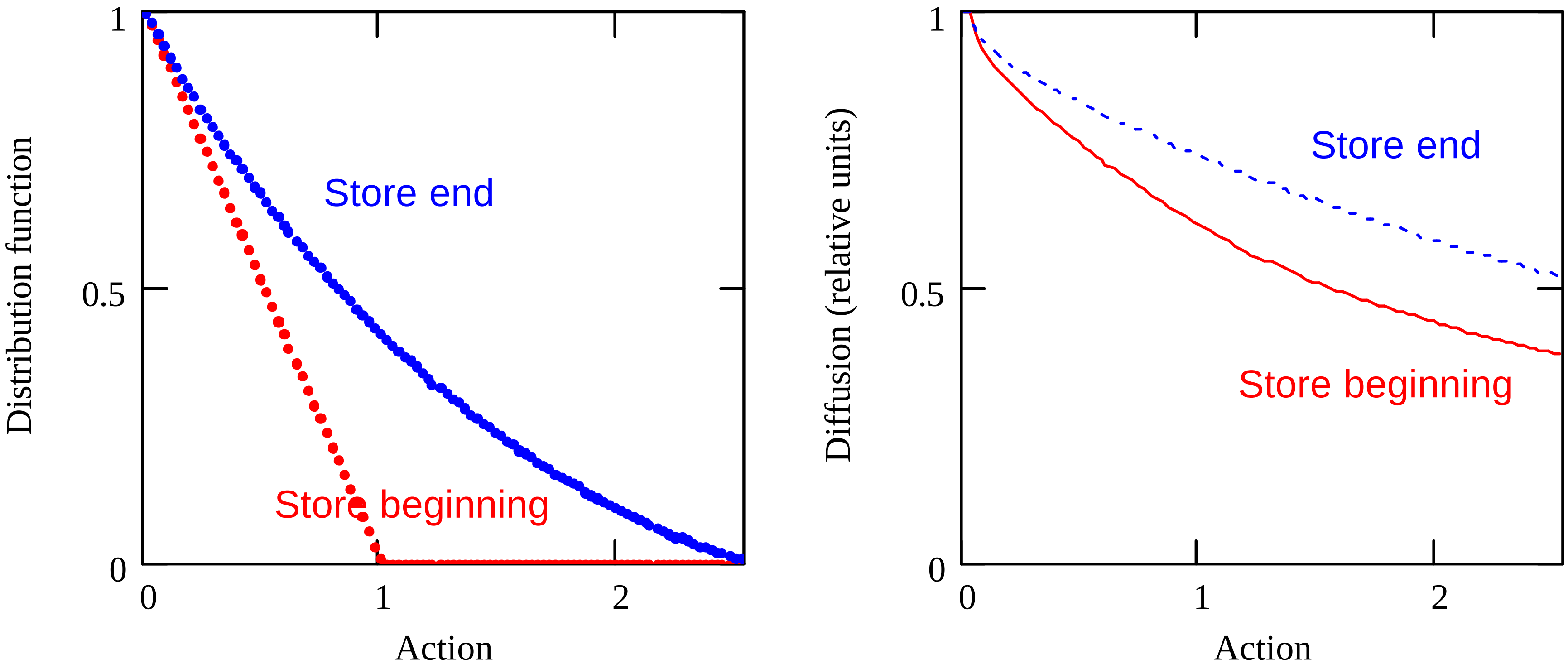
- That yields the following conclusions
  - There is "instant" luminosity loss due rebuilding of the distribution function ($<10^6$ turns)
  - Resonances do not produce significant diffusion amplification for this WP

Further Improvements of the diffusion model

- Comparison of measurements and predictions of the diffusion model should allow to observe more clearly the beam-beam effects
- Obtaining model parameters from independent measurements
  - Direct beam-based measurements of Tevatron vacuum
  - More accurate measurements of RF noise
- Accurate simulation of IBS and other diffusion mechanisms
  - Standard IBS theory implies gaussian distribution and is not self-consistent
  - Solving integro-differential equations for longitudinal and transverse distributions with the measured initial distribution functions ($v_{||} \ll v_{\perp}$). For longitudinal distribution:

$$\frac{\partial f}{\partial t} = \tilde{A}\int_0^{\infty} W(I,I')\left(f(I',t) - f(I,t)\right)\mathrm{d}I'$$

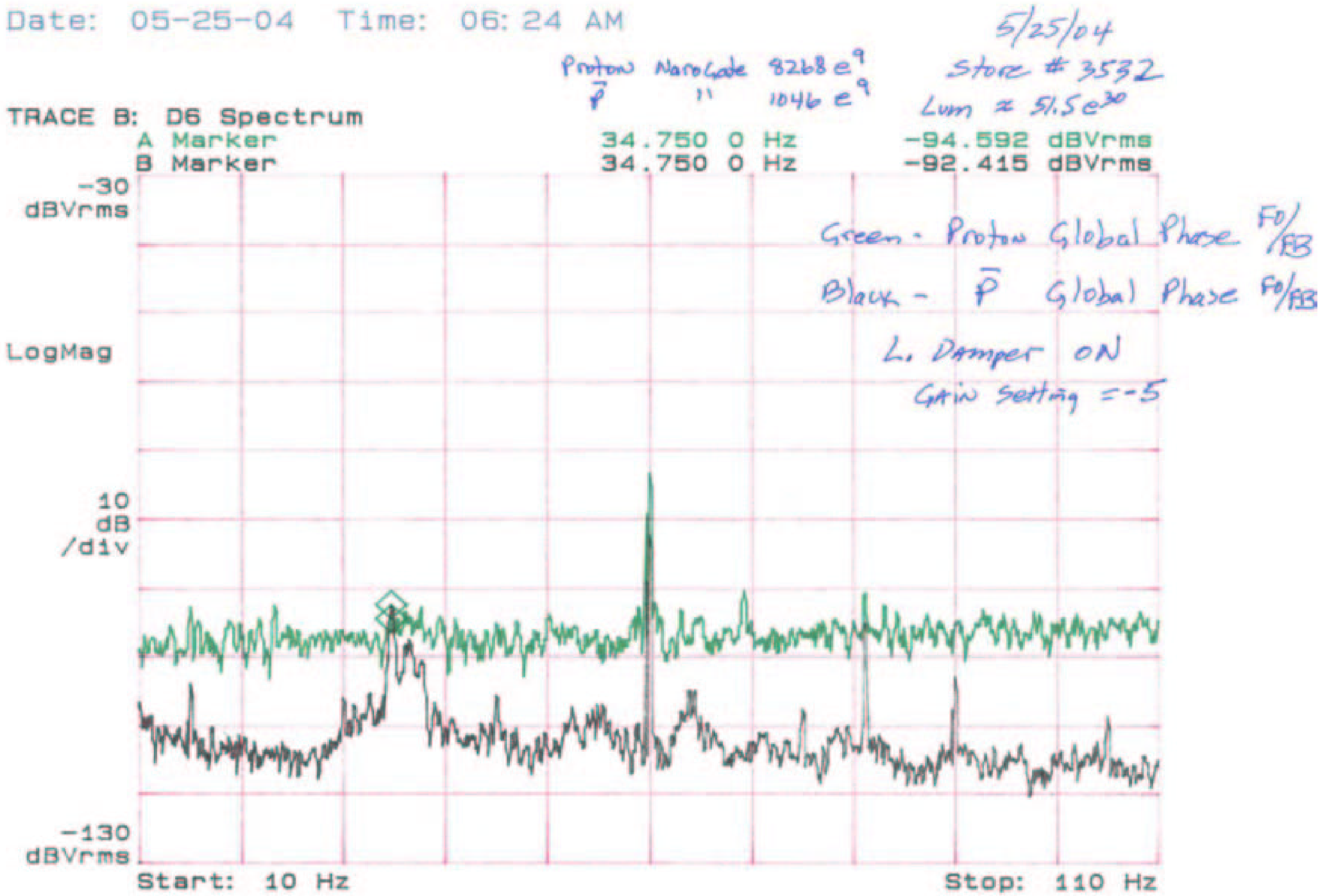
where the kernel is $W(I,I') = \frac{2ww'}{p}\int_0^{\min(a,a')} \frac{dz}{pp'} n(z)\left[\frac{1}{\left|p-p'\right|^3} + \frac{1}{\left|p+p'\right|^3}\right]$



Dependence of longitudinal distribution function and longitudinal diffusion on action for IBS

- The longitudinal diffusion depends on the action and the distribution function

RF noise



- Microphonics - cavity mechanical resonances are at synchrotron frequency
  - Phase feedback suppresses microphonics by more than 20 Db
- Longitudinal damper is too noisy
  - Damper “white” noise hides mechanical resonances

RF noise (continue)

- Noise affects nonlinear oscillator

$$\ddot{x} + \Omega_s^{\,2}\sin(x - y(t)) = 0 \quad \Rightarrow \quad \ddot{x} + \Omega_s^{\,2}\sin(x) = \Omega_s^{\,2}\cos(x)y(t)$$

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial I}\left(I\,\frac{D(I)}{w(I)}\frac{\partial f}{\partial I} \right)$$

- General case

$$D(I) = 2p\,\Omega_s^{\,2}\sum_{n=0}^{\infty} C_n(I)P\big(nw(I)\big)$$

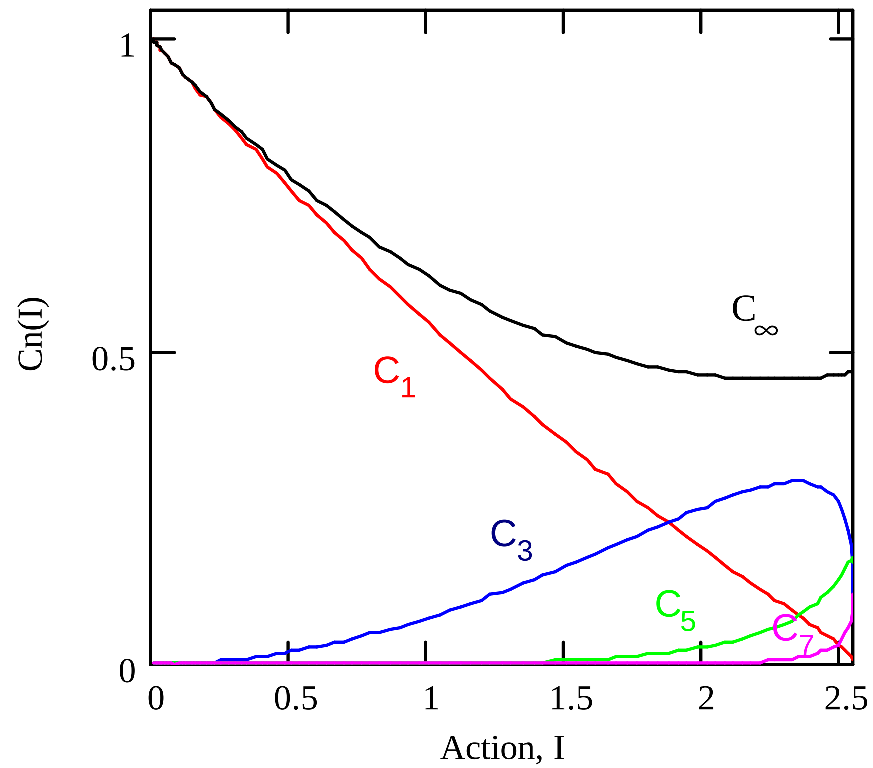
where the spectral density is normalized as

$$\overline{y(t)^2} = \int_{-\infty}^{\infty} P(w)dw$$

- For "white" noise, $P(w) = P_0$

$$D(I) = 2p\Omega_s^{\,2}P_0C_\infty(I) \quad ,$$

$$C_\infty(I) = \sum_{n=0}^{\infty} C_n(I)$$

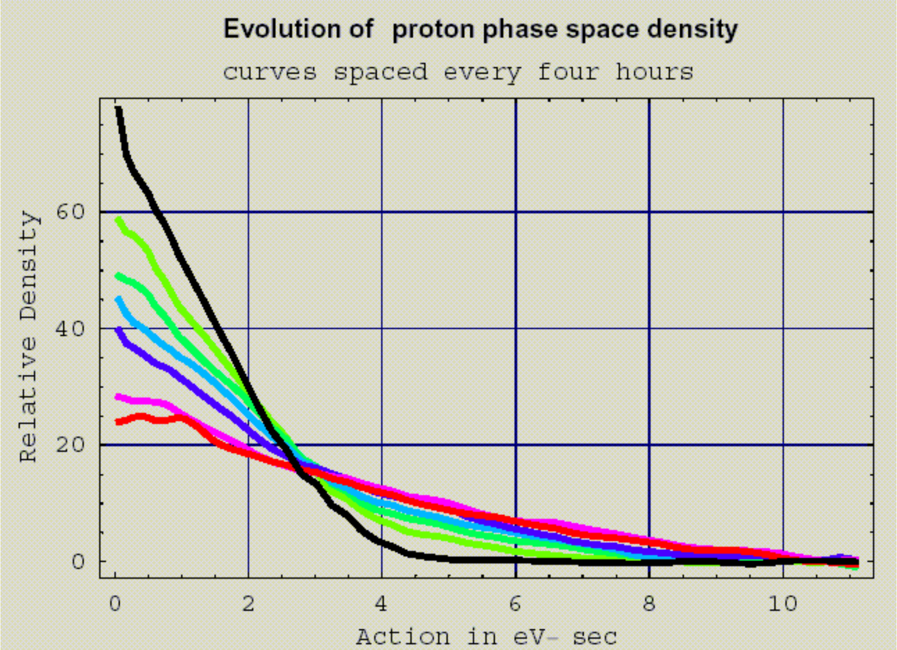


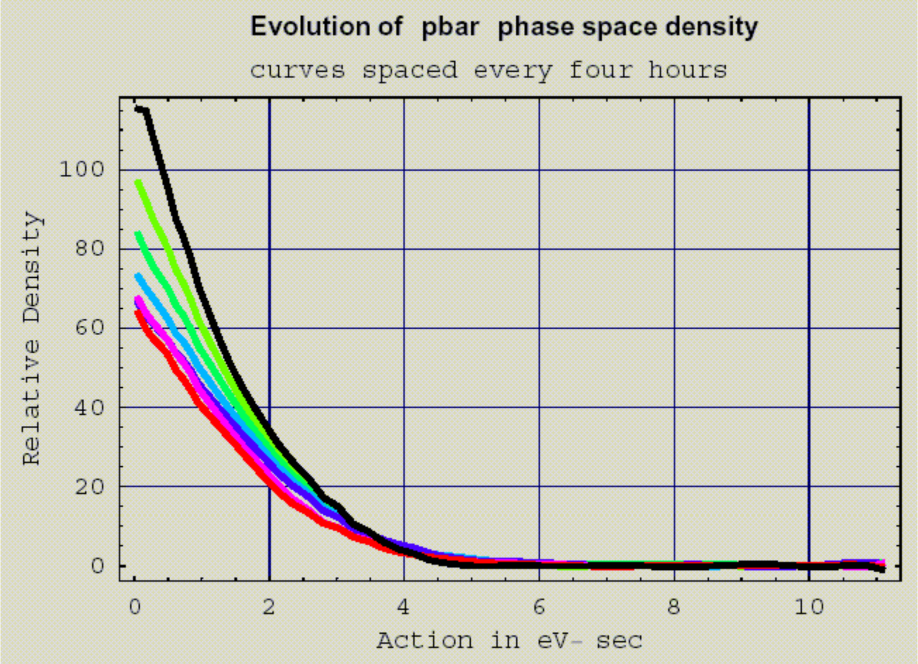
For all even n, $C_n(I) = 0$

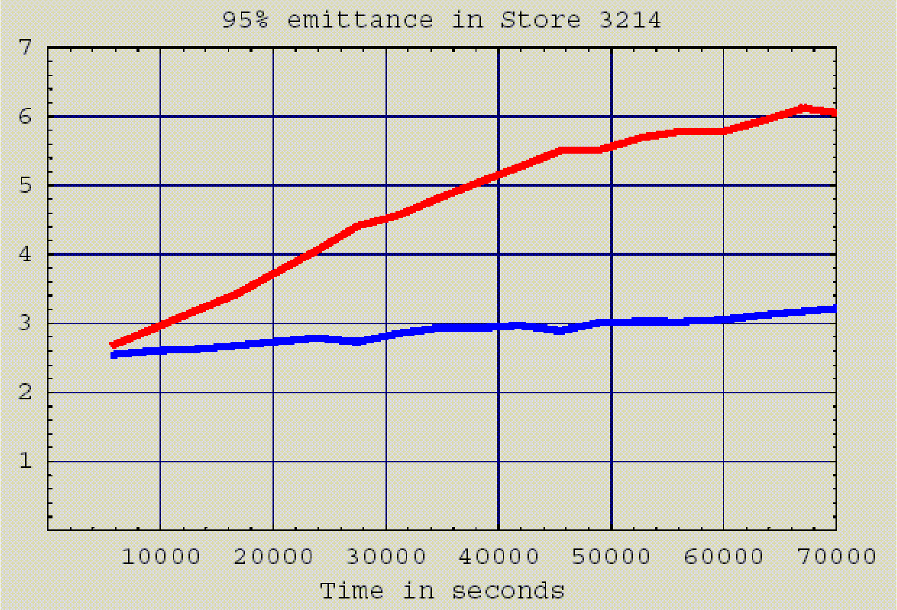
Improvements in diagnostics

- Beam emittance
  - Better optics knowledge $\Rightarrow$ better accuracy of emittance measurements
- Betatron tunes for every bunch
  - Old Schottky monitor (21 MHz) – all proton and pbar bunches together
  - New Schottky monitor (1.5 GHz) – each bunch separately but insufficient accuracy ($\sim 10^{-3}$)
  - Started to build "Another" tune monitor – high resolution (~0.1 μm per turn) turn-by-turn and bunch-by-bunch BPM
    - Recent measurements yield ~0.1 μm beam motion at betatron lines (2 orders of magnitude above Schottky noise)
- Chromaticity for every bunch during collisions
  - New Schottky monitor
- Longitudinal distribution function
  - Working off-line, will be operational soon

Dynamics of beam distribution in the longitudinal phase space







- Distribution functions are computed from signals of the resistive wall monitor (SBD)
- Entire bucket of 4.2 eV·s is filled at injection
- There is no long. emittance growth during acceleration
- Entire proton bucket of 10.7 eV·s is filled at the end of store

♦ ***Conclusions***

- There were a number of problems corrected in Tevatron during 3 years of Run II commissioning and operation
- Further luminosity growth will be mainly related with increase of pbar production
  - pbar intensity growth has to be supported by Tevatron
- Questions to the beam-beam effects study
  - Other WP (close to 2/3 (SPS), close to half integer)?
  - Correction of phase advance between IPs
  - Operation with 30 bunches versus present 36 bunches
- Other ways to mitigate beam-beam effects
  - Optimization of helical orbits for all stages
  - Increasing of HV separator strength and installation of new separators
  - On-line tune measurements and the tune feedback
  - Bunch length reduction
- Instrumentation
  - Precise tune measurements for every bunch

Interaction with Residual Gas (backup transparency)

Beam lifetime

$$\boxed{t_{scat}^{-1} = \frac{2pcr_p^2}{g^2 b^3}\left(\sum_i n_i Z_i (Z_i+1)\right)\left(\frac{\overline{b_x}}{e_{mx}} + \frac{\overline{b_y}}{e_{my}}\right) + \sum_i n_i s_i cb}$$

where: $\overline{b_{x,y}} = \frac{1}{C}\int b_{x,y}\,ds \quad \approx \quad 70\ \mathrm{m}$

e_{mx}, e_{my} – acceptances are chosen to be $6^2 \cdot 20$ mm mrad

- ♦ Average vacuum is adjusted to match the beam lifetime and the emittance growth rate for small intensity beam, $P=1\cdot10^{-9}$ Torr of N_2 equivalent
  - ➢ Coulomb scattering (~6000 hour)
  - ➢ Nuclear absorption **(~400 hour)**
  - ➢ Total gas scattering lifetime **(~380 hour)**
  - ➢ Gas composition used in the simulations

Gas	H_2	CO	N_2	C_2H_2	CH_4	CO_2	Ar
Pressure [nTorr]	1.05	0.18	0.09	0.075	0.015	0.09	0.15

Emittance growth time due to gas scattering

$$\boxed{\frac{de_{x,y}}{dt} = \frac{2pcr_p^2}{g^2 b^3}\left(\sum_i n_i Z_i (Z_i+1) L_{C_i}\right)\overline{b_{x,y}}} \quad \Rightarrow \quad \frac{de_x}{dt} \approx \frac{de_y}{dt} \approx 0.2\ \text{mm mrad/hour}$$

- Beam based measurements of vacuum were carried out in July 2003. Analysis will follow.

Scattering in IP (backup transparency)

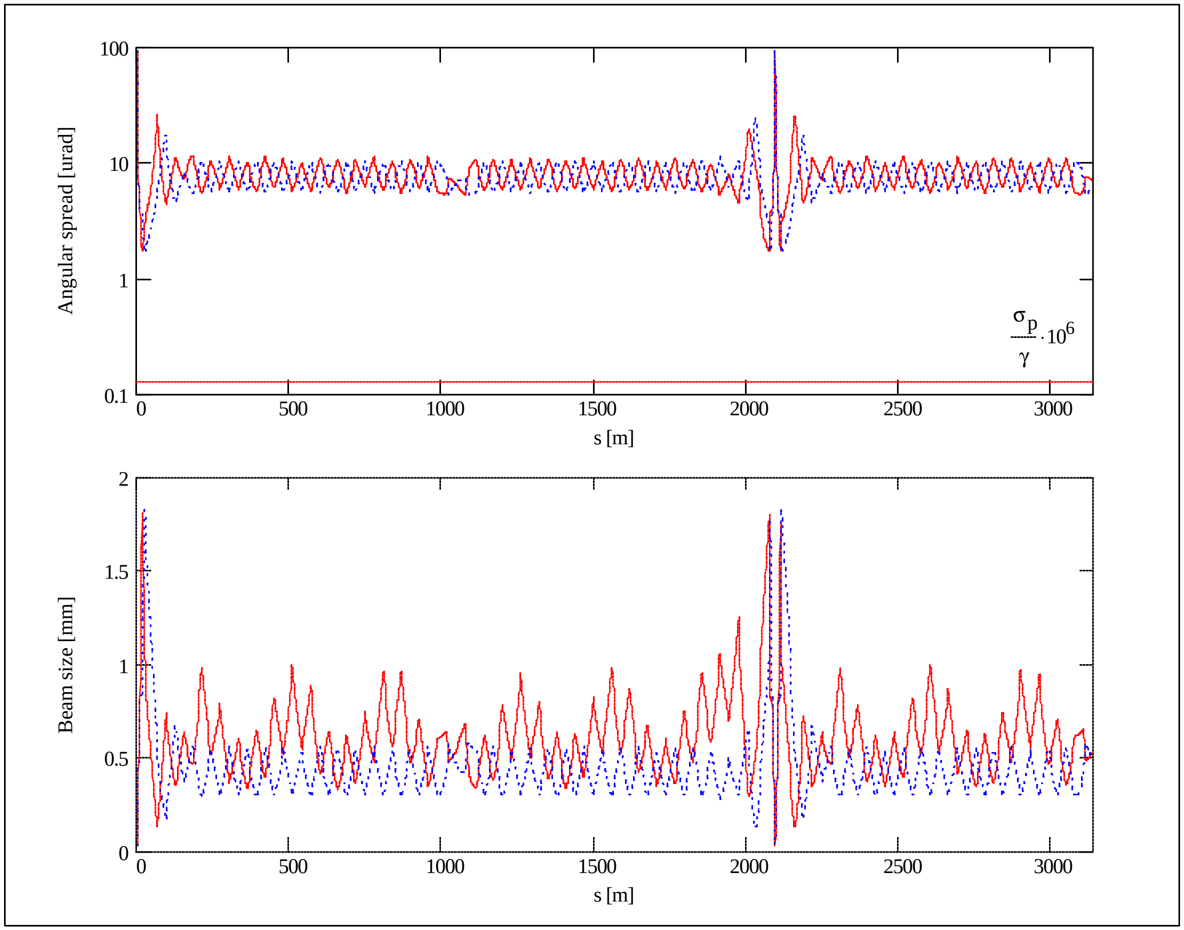
- Nuclear interaction
  - Main mechanism for loss of antiprotons
  - $p-\bar{p}$ cross-section ~ 69 mbarn
    - Inelastic – 60 mbarn
    - Elastic – 15 mbarn
      - 40% scatters within the beam (3σ)
- Electromagnetic scattering
  - Emittance growth

$$\frac{d\varepsilon_{x,y}}{dt} = \frac{4 r_p^{\;2} N L_{bb} f_0}{\gamma^2 \beta^3 \sqrt{\left(\varepsilon_{px} + \varepsilon_{py}\right)\left(\varepsilon_{ax} + \varepsilon_{ay}\right)}}$$

  - $d\varepsilon/dt \approx$ 0.01 mm mrad and is negligible in comparison with gas scattering

Intrabeam Scattering (backup transparency)

- Pancake distribution function allows one to use simple IBS formulas
- Integration over Tevatron lattice was carried out and results were compared to the smooth lattice approximation
  - Comparison yielded coincidence within 10%
- Therefore the smooth lattice approximation has been used for IBS to simplify the model



- The following corrections has been taken into account
  - Bunch length correction due to non-linearity of longitudinal focusing
  - Average dispersion and dispersion invariant, A_x, were calculated using lattice functions

Intrabeam Scattering (Continue)

Emittance growth rates for pan-cake gaussian distribution

$$\frac{d}{dt}\left(q_{\parallel}{}^{2}\right) \equiv \frac{d}{dt}\left(\frac{\overline{p_{\parallel}{}^{2}}}{p}\right) = \frac{1}{4\sqrt{2}} \frac{e^{4} N_{i} L_{C} \Xi_{\parallel}\left(q_{x}, q_{y}\right)}{m_{p}{}^{2} c^{3} g_{i}{}^{3} b_{i}{}^{3} s_{x} s_{y} s_{s} \sqrt{q_{x}{}^{2} + q_{y}{}^{2}}} \quad ,$$

$$\frac{de_{x}}{dt} = (1-k)\left\langle A_{x} \frac{dq_{\parallel}{}^{2}}{dt} \right\rangle_{s} \quad ,$$

$$\frac{de_{y}}{dt} = k\left\langle A_{x} \frac{dq_{\parallel}{}^{2}}{dt} \right\rangle_{s}$$

where

$$\Xi_{\parallel}(x,y) \approx 1 + \frac{\sqrt{2}}{p} \ln\left(\frac{x^{2}+y^{2}}{2xy}\right) - 0.055\left(\frac{x^{2}-y^{2}}{x^{2}+y^{2}}\right)^{2},$$

$$s_{x} = \sqrt{e_{x} b_{y} + D^{2} q_{\parallel}{}^{2}}, \quad s_{y} = \sqrt{e_{y} b_{y}}, \quad q_{x} = \sqrt{\frac{e_{x}}{b_{x}}\left(1 + \frac{\left(D' b_{x} + a_{x} D\right)^{2} q_{\parallel}{}^{2}}{e_{x} b_{x} + D^{2} q_{\parallel}{}^{2}}\right)}, \quad q_{y} = \sqrt{e_{y}/b_{y}}$$

$$A_{x} = \left\langle \frac{D^{2} + \left(D' b_{x} + a_{x} D\right)^{2}}{b_{x}} \right\rangle\Bigg|_{s}$$

k – coupling coefficient (measurements yield that presently k ~ 0.4)

A_x = 19.7 cm, b_x=b_y = 48.5 m, D = 2.84 m

Beam Evolution in Longitudinal Degree of Freedom for constant diffusion (backup transparency)

- Diffusion mechanisms
  - IBS
    - Multiple and single scattering
  - RF noise
    - Phase noise
    - Amplitude noise
- Diffusion differently depends on action for all three mechanisms

- The first iteration of the model solved diffusion equation in a sinusoidal potential well under constant diffusion, $D(I) = D,$

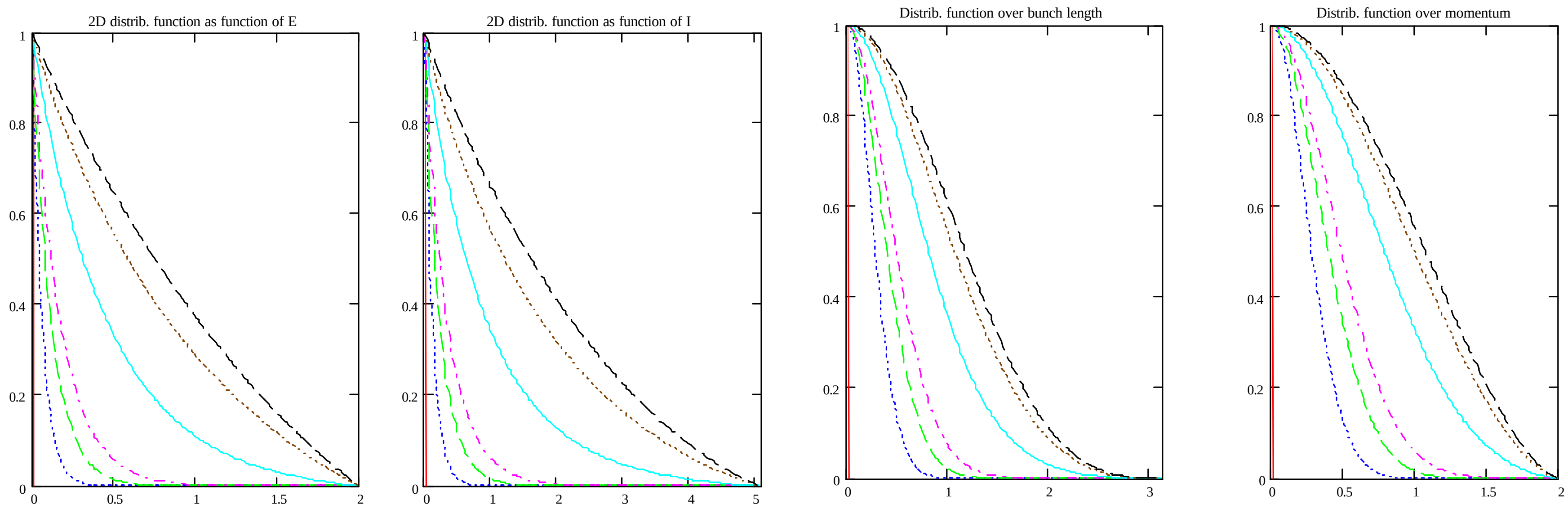
$$\frac{\partial f}{\partial t} = D\frac{\partial}{\partial I}\left(\frac{I}{dE/dI}\frac{\partial f}{\partial I}\right)$$

where

$$E = \frac{p^2}{2} + \Omega_s{}^2\left(1 - \cos f\right) \qquad , \qquad I = \frac{1}{2p}\oint p\,df$$

  - Equation is solved numerically for initial distribution f(I) = δ(I)
  - The boundary condition $f(I) = 0$ at the RF bucket boundary is used

Beam Evolution in Longitudinal Degree of Freedom (continue)

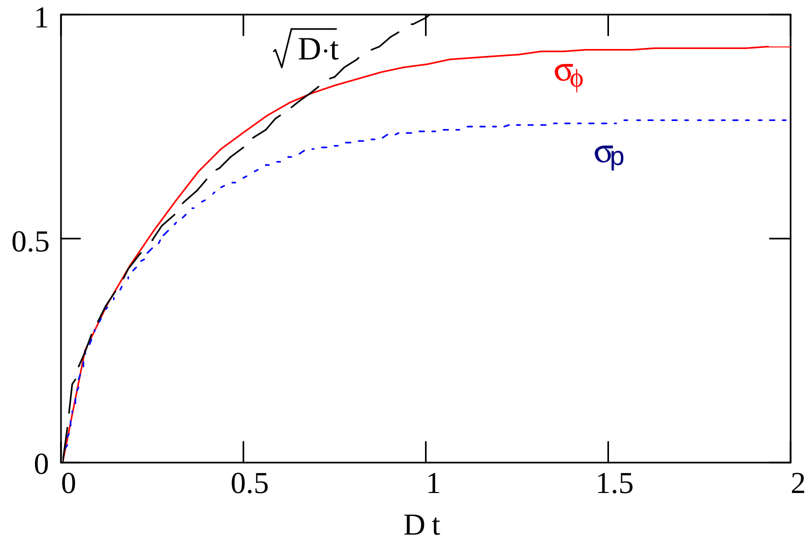


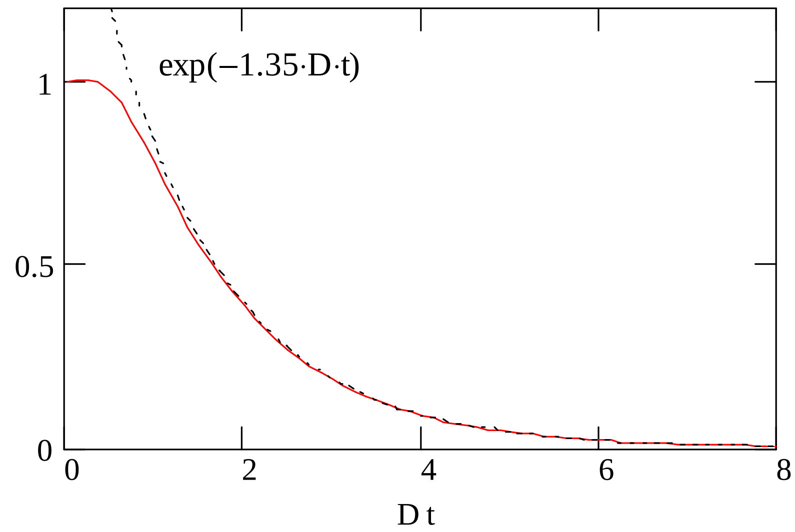
$f(I(E))$ $\qquad$ $f(I)$ $\qquad$ $f_f(f)$ $\qquad$ $f_p(p)$

Distribution functions as functions of the beam energy, action, longitudinal coordinate and the particle momentum deviation

where: $\int_0^{I_{max}} f(I)\,dI = 1$, $\quad f_f(f) = \int_{-p_{max}(f)}^{p_{max}(f)} f(I(f,p))\,dp$, $\quad f_p(p) = \int_{-f_{max}(p)}^{f_{max}(p)} f(I(f,p))\,df$.

Beam Evolution in Longitudinal Degree of Freedom (continue)





- Asymptotic behavior
  - Shape of distribution function does not depend on time
  - Exponential decay of beam intensity

Beam Evolution in Longitudinal Degree of Freedom (continue)

To find compromise between completeness and simplicity of the model the following approximate relations were deduced from the numerical solution:

$$s_s \approx \Gamma_s s_{\Delta p/p}\left(1+\frac{1}{4}\left(\frac{2s_{\Delta p/p}}{\Delta P/P|_{sep}}\right)^2+\frac{1}{6}\left(\frac{2s_{\Delta p/p}}{\Delta P/P|_{sep}}\right)^3\right)$$

$$\frac{1}{N}\frac{dN}{dt} \approx \frac{2.425\,(2p s_s)^7}{l_{RF}^{\;7}+1.65\,(2p s_s)^7}\left(\left(\frac{2p\Gamma_s}{l_{RF}}\right)^2 \left.\frac{d\left(s_{\Delta p/p}^{\;2}\right)}{dt}\right|_{IBS}+\left.\frac{d\left(s_f^{\;2}\right)}{dt}\right|_{RF}\right)$$

$$\left.\frac{d\left(s_{\Delta p/p}^{\;2}\right)}{dt}\right|_{total} \approx \left(1-\left(\frac{2s_{\Delta p/p}}{0.765\Delta P/P|_{sep}}\right)^5\right)\left(\left.\frac{d\left(s_{\Delta p/p}^{\;2}\right)}{dt}\right|_{IBS}+\left(\frac{l_{RF}}{2p\Gamma_s}\right)^2\left.\frac{d\left(s_f^{\;2}\right)}{dt}\right|_{RF}\right)$$

where $\Gamma_s = \left(a_M - 1/g^2\right) q\, l_{RF} / (2p n_s)$ is the parameter of longitudinal focusing

Beam Evolution in Longitudinal Degree of Freedom (continue)

The bunch lengthening due to RF phase noise

- At small amplitude the bunch lengthening due to RF phase and amplitude noise is determined by its spectral density at synchrotron frequency,

$$\left.\frac{d\left(s_f{}^2\right)}{dt}\right|_{RF} = p\,\Omega_s{}^2\left(P_f\left(\Omega_s\right) + \frac{1}{2}s_f{}^2 P_A\left(2\Omega_s\right)\right) \quad ,$$

where the spectral density of RF phase noise is normalized as

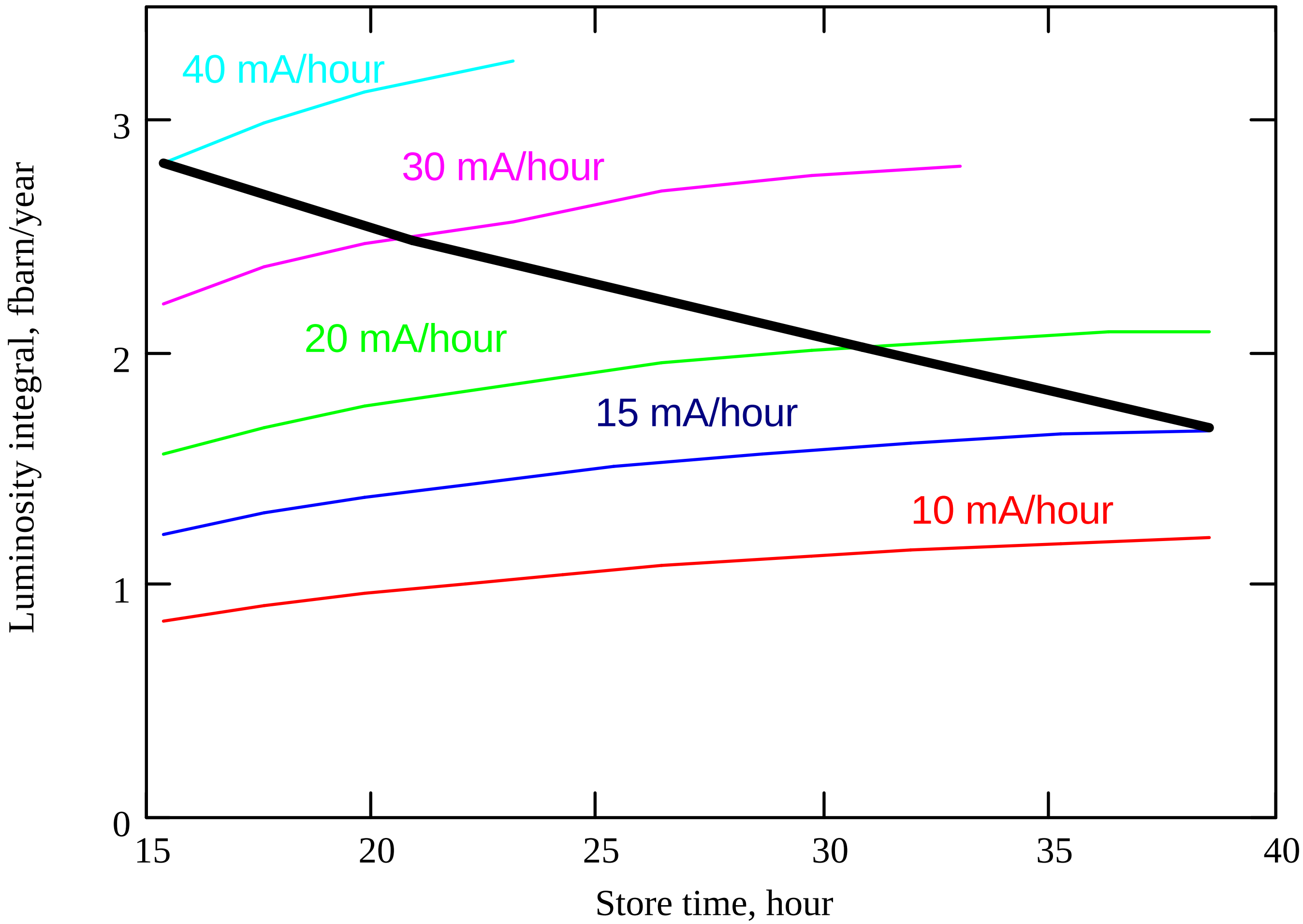
$$\overline{df_{RF}{}^2} = \int_{-\infty}^{\infty} P_f\left(w\right)dw \quad , \quad \frac{\overline{dA_{RF}{}^2}}{A_{RF}{}^2} = \int_{-\infty}^{\infty} P_A\left(w\right)dw$$

- Spectral density and bunch lengthening measurement are in decent agreement, and they yield that

$$P_{ff}\left(\Omega_s/2p\right) = 4pP_f\left(\Omega_s\right) \approx 5\cdot 10^{-11} \quad \mathrm{rad}^2/\mathrm{Hz}$$

$$\left.\frac{d\left(s_f{}^2\right)}{dt}\right|_{RF} \approx 2200 \quad \mathrm{mrad}^2/\mathrm{hour}$$

Dependence of luminosity on p-bar production rate (backup trasp.)



Dependence of luminosity integral per year on the store time for different antiproton production rates. Thick solid line shows where intensity of antiproton beam reaches $1.35\cdot10^{11}$ per bunch.

Accurate treatment of RF noise effects (backup transparency)

- RF phase noise effect on a nonlinear oscillator

$$\ddot{x} + \Omega_s^{\,2} \sin(x - y(t)) = 0 \quad \Rightarrow \quad \ddot{x} + \Omega_s^{\,2} \sin(x) = \Omega_s^{\,2} \cos(x) y(t)$$

Hamiltonian - $H = \frac{p^2}{2} + 2\Omega_s^{\,2} \left(\sin \frac{x}{2} \right)^2 - \Omega_s^{\,2} y(t) \sin x$

Action - $I = \frac{1}{2p} \oint p dx$

Frequency - $w \equiv w(I) = 2p \left(\oint \frac{dx}{p} \right)^{-1}$

Then the hamiltonian in the action-phase variables

$$H = wI - \Omega_s^{\,2} y(t) \sin\left(x(I, q) \right)$$

Corresponding motion equations -

$$\dot{I} = -\frac{\partial H}{\partial q} = \Omega_s^{\,2} y(t) \cos\left(x(I,q) \right) \frac{p(I,q)}{w}$$

$$\dot{q} = \frac{\partial H}{\partial I} = w - \Omega_s^{\,2} y(t) \frac{\partial}{\partial I} \left(\cos\left(x(I,q) \right) \right)$$

Solution in the first order of perturbation theory

$$dI(t) = \Omega_s^{\,2} \int_0^t y(t) \cos\left(x(I, wt + q_0) \right) \frac{p(I, wt + q_0)}{w}$$

That yields

$$\overline{dI(t)^2} = \frac{\Omega_s^{\,4}}{w^2} \int_0^t dt' \int_0^t dt'' \overline{y(t') y(t'')} \left\langle \cos\left(x(I, wt' + q_0) \right) p(I, wt' + q_0) \cos\left(x(I, wt'' + q_0) \right) p(I, wt'' + q_0) \right\rangle_{q_0}$$

Taking into account that: $K(t) = \overline{y(t) y(t+t)} = \int_{-\infty}^{\infty} P(w) e^{iwt}\, dw$

For $t \to \infty$ we obtain

$$\frac{d}{dt}\overline{dI^2} = 2p \frac{\Omega_s^{\,4}}{w^2} \sum_{n=-\infty}^{\infty} P(nw) \int_{-p/w}^{p/w} dt \int_{-p/w}^{p/w} dt\, e^{inwt} \cos\left(x(I,wt)\right) p(I,wt) \cos\left(x(I,wt)\right) p(I,wt)$$

We define the diffusion coefficient using the following form of diff. equation

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial I}\left(I \frac{D(I)}{w(I)} \frac{\partial f}{\partial I} \right)$$

Comparing the smearing of δ-functional distribution, $d(I - I_0)$, we finally obtain

$$D(I) = \frac{w}{I}\frac{d}{dt}\overline{dI^2} = 2p\Omega_s^{\,2} \sum_{n=0}^{\infty} C_n(I) P(nw) \quad ,$$

$$C_n(I) = \frac{2\Omega_s^{\,2} w}{I} \left| \oint \frac{dx}{2p} \cos x\, e^{iwt(x)} \right|^2$$

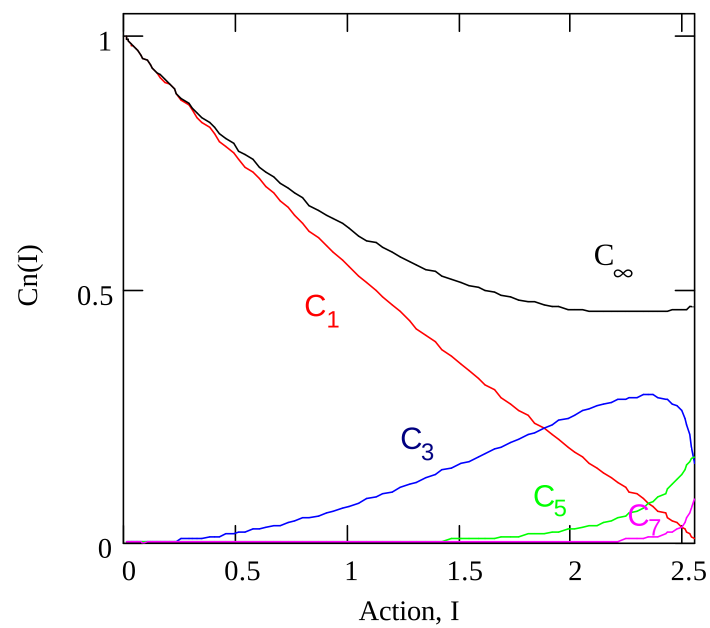
$$t(x) = \int_0^x \frac{dx}{\sqrt{2\Omega_s^{\,2}\left(\cos x - \cos\left(a(I)\right)\right)}}$$

where $a(I)$ is the oscillation amplitude.

For white noise, $P(w) = P_0$, it yields

$$D(I) = 2p\Omega_s^{\,2} P_0 C_\infty(I)$$

$$C_\infty(I) = \sum_{n=0}^{\infty} C_n(I) = \frac{4\Omega_s^{\,3}}{pI} \int_0^{a(I)} dx \cos^2(x) \sqrt{\frac{E}{2\Omega_s^{\,2}} - \sin^2\frac{x}{2}}$$



For all even n, $C_n(I) = 0$

Accurate IBS Treatment for Long.Degree of Freedom (backup)

- Simultaneous treatment of single and multiple scattering for longitudinal distribution (compare to IBS formulas at page 42):

$$\boxed{\frac{\partial f}{\partial t} = \tilde{A}\int_0^\infty W(I,I')\big(f(I',t) - f(I,t)\big)\mathrm{d}I'}$$

$$\tilde{A} = p^2\sqrt{\frac{p}{2}}\,\frac{\left(a - 1/g^2\right)e^4 N_i \Theta_{\|}\left(q_x, q_y\right)}{\Omega_s e V_0 m_p c g_i^{\,2} b_i C s_x s_y \sqrt{q_x^{\,2} + q_y^{\,2}}}$$

$$W(I,I') = \frac{2ww'}{p}\int_0^{\min(a,a')} \frac{dx}{pp'} n(x)\left[\frac{1}{\left|p - p'\right|^3} + \frac{1}{\left|p + p'\right|^3}\right] \xrightarrow{E' \geq E}$$

$$\frac{ww'}{p(E - E')^3}\left[(E - E')\int_0^a n(x)\frac{dx}{p} + 2\int_0^a n(x)\,p\,dx\right] \quad .$$

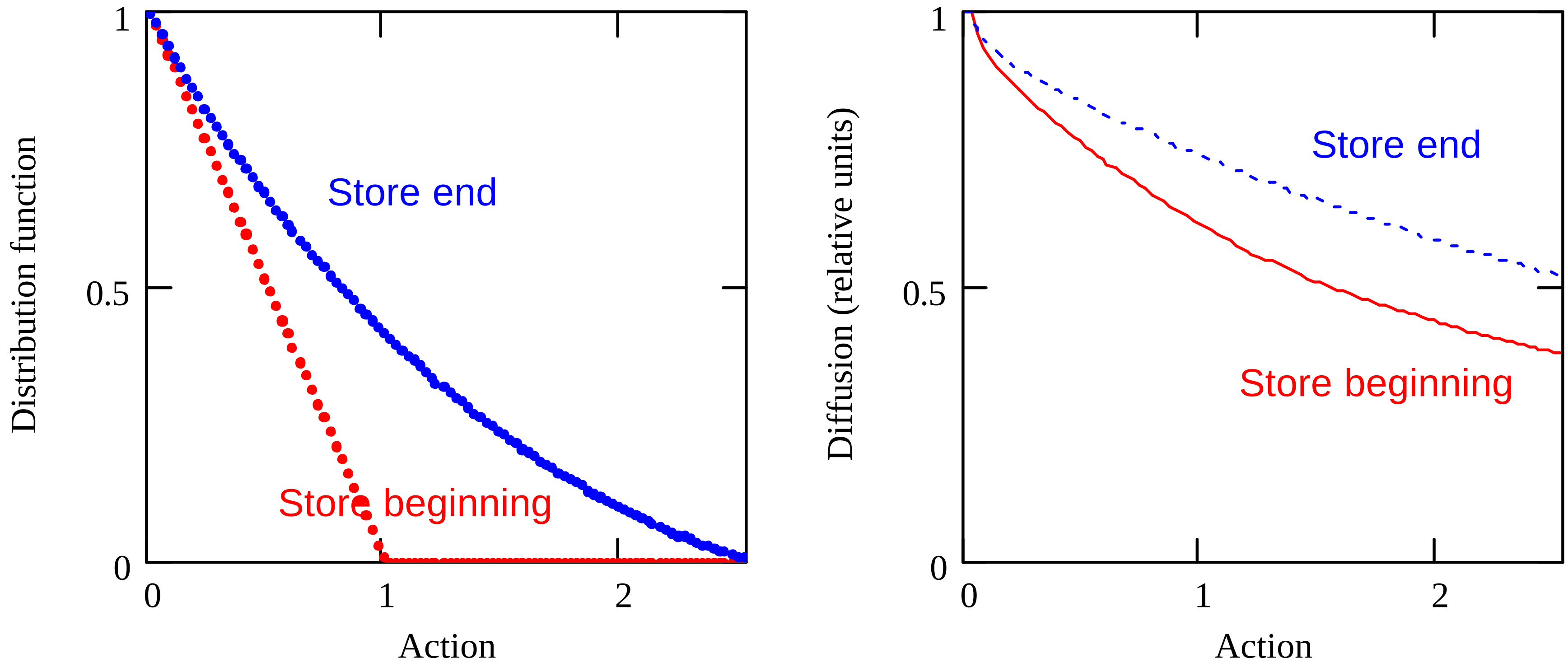
The kernel is symmetric: $W(I,I') = W(I',I)$,

The kernel divergence needs to be limited at the minimum action change corresponding to the maximum impact parameter

$n(x)$ is the long. particle density, $\int_{-p}^{p} n(x)dx = 1$, and $x \in [-p, p]$ - long. coordinate

V_0 is the RF voltage, C is the ring circumference, and

$a \equiv a(I)$ is the motion amplitude



Dependence of longitudinal distribution function and longitudinal diffusion on the action for IBS

IBS at coupled lattice for flat-distribution function (backup)

- For coupled motion the eigen-vectors can be parameterized as

$$\mathbf{v}_1 = \begin{bmatrix} \sqrt{b_{1x}} \\ -\dfrac{i(1-u)+a_{1x}}{\sqrt{b_{1x}}} \\ \sqrt{b_{1y}}e^{in_1} \\ -\dfrac{iu+a_{1y}}{\sqrt{b_{1y}}}e^{in_1} \end{bmatrix} \quad , \quad \mathbf{v}_2 = \begin{bmatrix} \sqrt{b_{2x}}e^{in_2} \\ -\dfrac{iu+a_{2x}}{\sqrt{b_{2x}}}e^{in_2} \\ \sqrt{b_{2y}} \\ -\dfrac{i(1-u)+a_{2y}}{\sqrt{b_{2y}}} \end{bmatrix}$$

where $b_{nx,ny}$ and $a_{nx,ny}$ are the beta- and alpha-functions, and parameters u, n_1 and n_2 are determined by the symplecticity conditions.

- Momentum change excites both hor. and vert. motions

$$\begin{bmatrix} D_x \\ D'_x \\ D_y \\ D'_y \end{bmatrix} \frac{\Delta p}{p} \equiv \mathbf{D}\frac{\Delta p}{p} = \mathrm{Re}\left(A_1\mathbf{v}_1 + A_2\mathbf{v}_2\right) = \mathbf{VA} \quad , \quad V = \left[\mathbf{v}'_1, -\mathbf{v}''_1, \mathbf{v}'_2, -\mathbf{v}''_2\right] \quad , \quad \mathbf{A} = \begin{bmatrix} A'_1 \\ A''_1 \\ A'_2 \\ A''_2 \end{bmatrix}$$

$$\mathbf{A} = \frac{\Delta p}{p}\mathbf{V}^{-1}\mathbf{D}$$

- Then the emittance growth is

$$\frac{de_1}{dt} = k_1 \frac{d}{dt}\overline{\left(\frac{\Delta p}{p}\right)^2} \quad , \quad \frac{de_2}{dt} = k_2 \frac{d}{dt}\overline{\left(\frac{\Delta p}{p}\right)^2}$$

where

$$k_{1,2} = \mathbf{D}^T \mathbf{B}_{1,2} \mathbf{D} \quad , \mathbf{B}_1 = \left(\mathbf{V}^{-1}\right)^T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{V}^{-1} \quad , \quad \mathbf{B}_2 = \left(\mathbf{V}^{-1}\right)^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{V}^{-1}$$

- Expressing matrix V through beta-functions we finally obtain

$$\mathbf{B}_1 = \begin{bmatrix} \dfrac{(1-u)^2 + a_{1x}{}^2}{b_{1x}} & a_{1x} & B_{1_{13}} & B_{1_{14}} \\ a_{1x} & b_{1x} & B_{1_{23}} & \sqrt{b_{1x} b_{1y}} \cos n_1 \\ B_{1_{13}} & B_{1_{23}} & \dfrac{u^2 + a_{1y}{}^2}{b_{1y}} & a_{1y} \\ B_{1_{14}} & \sqrt{b_{1x} b_{1y}} \cos n_1 & a_{1y} & b_{1y} \end{bmatrix}$$

$$B_{1_{13}} = \frac{\left(u(1-u) + a_{1x} a_{1y}\right)\cos n_1 + \left(a_{1y}(1-u) - a_{1x} u\right)\sin n_1}{\sqrt{b_{1x} b_{1y}}}$$

$$B_{1_{14}} = \sqrt{\frac{b_{1y}}{b_{1x}}}\left(a_{1x} \cos n_1 + (1-u)\sin n_1\right)$$

$$B_{1_{23}} = \sqrt{\frac{b_{1x}}{b_{1y}}}\left(a_{1y} \cos n_1 - u \sin n_1\right)$$

$$
\mathbf{B}_2 = \begin{bmatrix} \dfrac{u^2 + a_{2x}{}^2}{b_{2x}} & a_{2x} & B_{2_{13}} & B_{2_{14}} \\ a_{2x} & b_{2x} & B_{2_{23}} & \sqrt{b_{2x} b_{2y}} \cos n_2 \\ B_{2_{13}} & B_{2_{23}} & \dfrac{(1-u)^2 + a_{2y}{}^2}{b_{2y}} & a_{2y} \\ B_{2_{14}} & \sqrt{b_{2x} b_{2y}} \cos n_2 & a_{2y} & b_{2y} \end{bmatrix}
$$

$$
B_{2_{13}} = \frac{\left(u(1-u) + a_{2x} a_{2y}\right) \cos n_2 + \left(a_{2x}(1-u) - a_{2y} u\right) \sin n_2}{\sqrt{b_{2x} b_{2y}}}
$$

$$
B_{2_{14}} = \sqrt{\frac{b_{2y}}{b_{2x}}} \left(a_{2x} \cos n_2 - u \sin n_2\right)
$$

$$
B_{2_{23}} = \sqrt{\frac{b_{2x}}{b_{2y}}} \left(a_{2y} \cos n_2 + (1-u) \sin n_2\right)
$$